THREE ESSAYS ON GROWTH AND THE WORLD ECONOMY

PhD Thesis presented by D. IÑAKI ERAUSKIN IURRITA

Supervised by Dr. F. JAVIER GARDEAZABAL MATÍAS

EUSKAL HERRIKO UNIBERTSITATEA
UNIVERSIDAD DEL PAÍS VASCO

ANALISI EKONOMIKOAREN OINARRIAK II SAILA
DEPARTAMENTO DE FUNDAMENTOS DEL
ANÁLISIS ECONÓMICO II

DONOSTIA, MARCH 2004

Acknowlegments

Once I have finished this thesis for the obtention of the Ph.D. degree I would like to thank different people for making it possible.

First and foremost, I have got a huge debt with my family, that has provided much love and support during all my life. Without them I am sure I would not write these words.

For all her true love and encouragement I deeply thank Elena. Cupid luckily arranged to meet us more than four years ago.

I am heartily pleased that Javier Gardeazabal agreed to be my supervisor seven years ago, since I have had the opportunity to meet, in addition to an outstanding economist, an excellent person. He has provided me with the optimal combination of very helpful suggestions and criticism reading carefully all the drafts, and warm encouragement throughout the writing of the thesis. I deeply thank him for all his help. In addition, I thank him for suggesting, together with my colleague Iñaki Peña, to write this thesis in English.

I am very grateful to both the former and the actual Dean of ESTE School of Management at the Universidad de Deusto (San Sebastian), Francisco Olarte and Victor Urcelay respectively. Since I started to work at ESTE in 1994 they have provided me with the necessary support at different stages of the thesis.

I thank very much deeply the former Chair of the Department of Economics at ESTE, Mikel Navarro, for giving me the chance of joining ESTE and for being so helpful whenever it was necessary.

Fernando de la Iglesia s.j. has provided me with much support and comprehension both as a colleague and as the former Chair of the Department of Economics for many years. I truly thank him for his help.

The support of the actual Chair of the Department of Economics, Mari Jose Aranguren, is gratefully acknowledged as well.

I thank all my colleagues at the Department of Economics and at the University of Deusto. I would like to mention specially Asier Minondo for all his help and encouragement, even reading and commenting the whole thesis, and Aitor Garmendia, and Iñaki Peña for their support throughout the writing of the thesis.

The staff of the Library, captained by Carmen Navarrete, has always been very helpful for obtaining the numerous references for the thesis. I would like to thank them all for doing a superb work.

I thank Jaume Ventura for very kindly providing me the data on which Chapter 4 (Essay 3) is based.

I would like to thank a person that, even though I have not had the pleasure to meet him, has been a guidance throughout these years. Anyone reading this thesis will very soon realize that Stephen Turnovsky´s superb extensive work has been fundamental for this thesis.

I thank Cruz Ángel Echevarría and Jesús Vázquez for providing me with numerous and very useful comments on earlier drafts of this thesis.

Suggestions and comments received from seminar participants at the XXVIII Simposio de Análisis Económico in Sevilla and at the "Brown Bag" seminars at the Departamento de Fundamentos del Análisis Económico II (Universidad del País Vasco) are gratefully acknowledged.

I thank all the professors and organizers that have taken part in the Programa de Doctorado en Economía (Análisis Económico y Economía Pública) organized by the departments of Fundamentos del Análisis Económico (Teoría Económica), Economía Aplicada II (Hacienda Pública y Derecho Fiscal) and Economía Aplicada III (Econometría y Estadística) at the Euskal Herriko Unibertsitatea-Universidad del País Vasco for their helpful guidance.

I want to thank Professor Philippe Bacchetta and Dr. Javier Coto-Martínez for writing the reports required by the University so that this thesis can be considered for a "Doctorado Europeo" mention.

I thank all the members of my thesis committee for the excellent disposition to be in the committee.

I thank heartfully Inma Muñoa and Maite Manterola for helping me to polish my written English.

I owe very much to my other family, my friends, for making my life much more lively and pleasant. Thank you!

I should note that I obviously remain the only responsible for the errors, omissions and misunderstandings in my thesis.

I apologize for any person I have forgotten to mention here.

To all of them my sincerest gratitude.

Eskerrik asko lagundu didazuen guztioi!

Iñaki Erauskin-Iurrita

Lasarte-Oria, Basque Country
March 10, 2004.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

The introduction of the thesis has three parts. First, we establish the general research objectives of this thesis. Additionally, we enumerate the questions we try to answer, and the general framework chosen in order to respond the questions. Second, we review briefly the literature that is necessary to reach our objectives. Finally, we describe the structure of the thesis and the specific objectives pursued in each chapter.

1.1 Research objectives

Two are the general research objectives of this thesis. The first general research objective is to analyze of the expenditure policy of the public sector and risk in a two-country world where the public sector provides a good that is either utility-enhancing or productivity- and volatility-enhancing. In addition, we obtain the optimal size of the public sector and we analyze whether more open economies should have a higher size of the public sector or not. The second general research objective is to analyze and empirically test the impact of transitory income shocks on the current account, based on an extension of the new rule of the intertemporal approach to the current account in a two-country world.

Thus, three are the specific questions we try to answer:

- How does the spending policy of the public sector and risk influence on key economic variables, such as consumption, the growth rate or welfare? How does openness influence on those key economic variables?

- Which is the optimal size of the public sector? Should more open economies be associated with a higher size of the public sector?

- Which is the impact of transitory income shocks on the current account in a two-country world? How does the theory fit with the empirical data?

The common framework of analysis we use to respond to those questions is a two-country stochastic AK growth model in continuous time. Several reasons have leaded us to choose this model. First, the AK approach can be very useful in certain situations. Thus, according to Aghion and Howitt (1998, p. 2), "In spite of its reduced-form representation of the process of knowledge accumulation, the AK formulation appears to be quite useful, especially when discussing government policies from an aggregate perspective". In the same vein, Turnovsky (2000, p. 423) argues that AK models are "well suited" in order to analyze macroeconomic policy in models motored by investment that generate endogenous growth. Second, the stochastic optimal control gives much more realism to economic analysis, since it can model measurement errors, omission of important variables, non-exact relationships, incomplete theories and other methodological complexities, allowing some variables to be random (stochastic) and introducing pure randomness through the white noise (Malliaris, 1987, p. 502). Third, working with stochastic models usually imply stronger limitations compared to other models. Thus Turnovsky (2000, p. 424) argues that " these stochastic models are generally tractable only if the technology is of a restrictive type, namely of the AK form". Fourth, Merton (1987 [1998, p. 628]) notes that "These [continuous-time] models frequently produce significantly sharper results that can be derived from their discrete-time counterparts". In the same direction, Turnovsky (1997, p. 325) notes "our main reason for this choice is that although continuous-time stochastic problems are tractable only under restrictive conditions, when these conditions are met, the solutions they yield are highly transparent, providing substantial insights into the characteristics of the equilibrium." Finally, since goods and capital markets are becoming increasingly integrated, we have considered that a two-country world economy is a convenient framework of analysis.

1.2 A brief review of the literature

We review briefly the most important recent developments in two specific strands of the economic literature, given the specific questions we aim to answer:

- The impact of the expenditure policy of the public sector and risk on economic growth, and
- The impact of transitory income shocks on the current account.

1.2.1 The role of the expenditure policy of the public sector and risk on economic growth

In this section first we briefly review the "three waves of interest in growth theory" [Solow (1994, p. 45)] in the past 60 years or so,[1] before embarking on the discussion about the role of public spending policy and risk on long term growth.

First, focusing on the theory of growth, the first wave of interest goes back to Harrod (1939, 1948) and Domar (1946, 1947). Both combine two of the elements of the Keynesian corpus (namely, the multiplier and the accelerator) to explain long term growth, assuming a fixed proportion-technology. As Solow (1956, p. 65) puts it, "The characteristics and powerful conclusion of the Harrod-Domar line of thought is that even for the long run the economic system is at best balanced on a knife-edge of equilibrium growth", which implies that in case the economy deviates from the knife-edge then the economy will suffer either from increasing unemployment or ongoing inflation.

The second wave of interest comes with the construction of the neoclassical model, pioneered by Solow (1956) and Swan (1956). The key element

[1] There are many excellent recent treatments on economic growth. See Barro and Sala-i-Martin (1995), Aghion and Howitt (1998), for example, for advanced treatments, and Gylfason (1999), Jones (2002), Sala-i-Martin (2000), and Van den Berg (2001), for example, for more asequible treatments. Turnovsky (2001, 2003) provide recent excellent overviews of investment-based growth models for an open economy and a closed economy, respectively. See Rostow (1990) for an exhaustive reference on the theorists of economic growth from David Hume to these days, but with almost no references to new growth theories. Burmeister and Dobell (1970), and Wan (1971) provide excellent treatments prior to the new growth theory.

of the model, as opposed to the Harrod-Domar model, is that the production function exhibits constant returns to scale, diminishing, albeit positive, marginal productivity of each factor of production, smooth elasticity of substitution between factors and it satisfies the Inada conditions (the marginal product of labor -or capital- approaches infinity as labor -or capital- tends to zero and it approaches zero as labor -or capital- tends to infinity). One of the main conclusions of the neoclassical model is that, in the absence of technological improvements, per capita variables do not change or, put another way, per capita growth vanishes, which implies that exogenous technological progress needs to be introduced for the model to generate long term growth. In addition, the neoclassical model shows the conditional convergence of each economy to its own steady state and that the speed of convergence is inversely related to the distance from the steady state.

The third wave of interest can be found in the advent of the endogenous growth models around mid-80s, leaded by Romer (1986, from his 1983 thesis) and Lucas (1988, from his 1985 Marshall Lectures). The enormous strength and impetus acquired by the research on economic growth recently is mainly due to the emergence of endogenous growth models. The fact is that the standard neoclassical growth theory based long term ongoing growth on the behavior of a exogenous variable such as technological improvement, which was not very appealing[2]. The new research based their new growth theories on the behavior of diverse endogenous variables: in these new models long run sustained growth is determined by endogenous variables. That is the reason why such models have become widely known as endogenous growth models. The most important property of endogenous growth models is that there are no diminishing returns to capital. They are usually divided into two groups (Turnovsky, 2000, p. 421). The first type of endogenous growth models is based on the fact that the main source of economic growth is the accumulation of private capital, which makes it very close to the neoclassical model. The most basic production function without diminishing returns that is able to generate endogenous growth in the first type of models is

[2] Two reasons were fundamentally behind the reactivation of endogenous growth models around mid-80s, according to de la Fuente (1992, p. 354): the technical difficulty of incorporating dynamic models into non-competitive market structures, on the one hand, and the change in priorities of interests of macroeconomists from business cycles towards long term growth, on the other hand.

$$Y = AK$$

where Y is the flow of production, A is the positive (constant) marginal physical product of capital and K is the stock of capital. This type of production function is commonly known as "AK function (or AK technology)"[3], and the model based on such a production function, AK model, or sometimes "linear-in-K" model (Valdés, 1999, p. 107). Even though the supposition of no diminishing returns to capital seems to be rather heroic, the growing literature has usually understood that K should be interpreted in a broad sense so as to include human as well as physical capital, following the pioneer work by Rebelo (1991), so that such a supposition may not be so heroic[4]. Around this type of production function diverse endogenous growth models have been constructed, models with physical and human capital, models with learning-by-doing and knowledge spillovers, models where the public sector influences on long term growth, etc. The second type of endogenous growth models is based on the fact that the main source of economic growth is the endogenous development of knowledge, or research or development. Our point of departure will be an AK model of endogenous growth (first type) where the public sector can influence significantly on the economic activity through spending.

Second, narrowing our analysis, we focus on the role of the expenditure policy of the public sector and risk on growth. It is not surprising that the economic profession have dedicated much effort recently to analyze the link between fiscal policy and economic growth since endogenous growth models provide some margin to improve the long run behavior of the economy. In contrast to endogenous growth models, in the Solow-Swan neoclassical

[3] The origins of this production function do not seem to be completely clear. Barro and Sala-i-Martin (1995, p. 39, footnote 12) think that the first economist that used this type of function was von Neumann in 1937. However, von Neumann´s article was published in 1938. Aghion and Howitt (1998, p. 24, footnote 18) attribute this type of function to Harrod (1939) and Domar (1946). Rebelo (1991, p. 507, footnote 6) points out that it dates back to Knight (1935, 1944) and Hagen (1942). Turnovsky (1997, p. 153; 2000, p. 422) goes back to Harrod (1939). Despite all that, it is usually attributed to Rebelo (1991) the incorporation of the AK linear production function to the recent developments in the field of endogenous growth (Sala-i-Martin, 2000, p. 51).

[4] Thus even though it seems to imply that labour is not used at all as a factor of production that is not true in fact.

growth model "conventional macroeconomic policy had no influence on long-run growth performance" (Turnovsky, 2003, p. 1). Even though fiscal policy includes many issues, such as public spending, taxation, deficit financing, etc., here we only focus on the impact of expenditure policy of the public sector on growth. Thus public spending could affect steady state output per worker, but not long run growth (except transitorily), in Solow-Swan or Ramsey type models. Therefore, the role of public spending was not considered. For example, it is remarkably meaningful that two leading textbooks on the theory of economic growth in the 70's have no references whatsoever to the role of government spending on economic growth[5]. It was Aschauer (1989) that "hit the magic button" (Gramlich, 1994, p. 1176) with his pioneer empirical analysis relating government spending on physical structures and productivity slowdown. Now, as Turnovsky (2000, p. 228) puts it, "This [Considering the spending of the public sector as productive] is becoming a topical issue and only recently has begun to receive analytical treatment by macroeconomists". The AK models have played a prominent role on the analysis of the impact of fiscal policy on growth. We refer to Turnovsky (1997, 2000) for two superb textbook modern treatments that dedicate part of them to these issues, and Zagler and Dürnecker (2003) for an extensive survey on the literature concerning the link between fiscal policy and economic growth.

Barro (1990) constructs the first model to analyze how government spending influences on growth in a deterministic AK growth closed economy, following two approaches basically. According to the first approach, public spending is introduced in the utility function, but it does not affect the marginal product of private capital. Therefore, a higher size of the public sector reduces unambiguously the growth rate, even though it may raise welfare. Turnovsky (1996) extends the basic model to a deterministic small open economy setting. In order to obtain a path of ongoing growth, the expenditure of the public sector must be tied either to private consumption or domestic wealth. Both rules have different implications. If government expenditure is tied to private consumption, then private consumption and domestic wealth grow at the same rate, which is different from the growth rate of domestic capital. Additionally, a higher size of the public sector reduces consumption-wealth ratio, but the rates of growth of wealth and capital do not change. Instead, if the expenditure of the public sector is linked to wealth, then a

[5]See Burmeister and Dobell (1970) and Wan (1971).

higher size of the public sector, even though it does not change the growth rate of capital, reduces the rate of accumulation of wealth. Then Turnovsky (1996, p. 60) shows that "once the parameters defining the expenditure rules are optimally chosen, the rules themselves used to define these policies cease to be important. Furthermore, precisely the same overall optimum emerges if one optimizes directly with respect to G, i.e. without postulating any specific form of expenditure rule". The different consequences of both spending policies (tied to private consumption and wealth) for optimal tax policy are considered as well. The result is that the optimal tax structure depends crucially on the expenditure rule the public sector follows.

The way in which risk is introduced into endogenous growth models follows basically Eaton (1981), where the assumption that stochastic productivity disturbances are proportional to some state variable (capital, wealth, etc.) results very convenient. However, public spending was neither utility- nor productivity-enhancing in his model. Turnovsky (1999) extends Barro's model in a stochastic small open economy setting, provided that public spending is utility-enhancing. Thus both the effectiveness and the size of the public sector in an open economy are compared to those in a closed economy. This issue is most relevant if we look at the current trend towards a higher interdependence in goods and capital markets. Thus Rodrik (1998) shows that economies that are more open to international trade have bigger governments and argues that it is due to the fact that government spending provides social insurance against external risk.[6] Instead, Turnovsky (1999, p. 901) points out that "in contrast to Rodrik, we find that the overall gain in domestic stability is likely to result from the export of domestic-source variability, rather than from sheltering the economy from foreign source instability.". Additionally, Turnovsky finds that the optimal size of the public sector in an open economy is higher than that in a closed economy, provided that the domestic economy holds positive assets of the foreign economy.

In the second approach formulated by Barro (1990) the spending of the public sector is assumed to be productive, following the pioneer empirical work by Aschauer (1989). Public spending is introduced, usually as a flow,

[6]However, Alesina and Wacziarg (1998) argue that the link between the size of the public sector and openness can be explained alternatively on the grounds that a higher size of the public sector is related to small economies (due to the economies of scale involved in the provision of public goods) and that small economies are usually more open to trade, so that country size is the variable that can account for the positive relation between the size of the public sector and the openness to trade.

in the production function, so that it alters the marginal physical product of private capital, capturing the positive effect of investment in physical structures. Productive spending raises the growth rate for "low" values of the size of the public sector. However, the growth rate falls eventually for "high" values of the size of the public sector. The basic results of Barro's model are, on the one hand, that the optimal size of the public sector is equal to the weight of public spending in the Cobb-Douglas type production function and, on the other hand, that the size of the public sector that maximizes welfare coincides with the size of the public sector that maximizes the growth rate. Then Turnovsky (1998) has analyzed the role of public spending in a stochastically growing closed economy incorporating congestion features. Since public spending, in addition to increasing productivity, increases volatility, then risk unambiguously reduces the optimal size of the public sector, as formulated by Barro (1990). In addition, risk introduces a divergence between the growth rate that maximizes welfare and that which maximizes growth. Turnovsky (1999) has extended the analysis in a stochastic small open economy. The basic conclusion formulated is that the optimal size of the public sector in an open economy is higher than that in a closed economy, provided that the domestic economy holds stocks of foreign assets, as in the utility-enhancing case.

Finally, we refer briefly to models where public spending is neither utility- nor productivity-enhancing, or even it is ignored, but which are very useful for our purposes. Obstfeld (1994) studies the impact of risk on growth and welfare in a multi-country world economy. Comparing the results of open economies with those of closed economies, the main conclusion is that "international risk-sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth is the attendant world portfolio shift from safe, but low-yield, capital into riskier, high-yield capital" (Obstfeld, 1994, p. 1326-27). Next, in a quite closely related paper, Turnovsky (1997, Ch. 11) constructs a risky two-country AK growth world economy, where public spending is "a real drain on the economy" (p. 338) and the size of the public sector is exogenously given. Three issues are analyzed within that framework: the influence of risk on the growth rate and welfare [as in Obstfeld (1994)], the equilibrium of the world economy and how it responds to changes in the stochastic structure, and the relationship between export instability and the growth rate. We find that the model set up by Turnovsky (1997, Ch. 11) provides a convenient framework to address the impact of public spending policy and risk on the world economy. In fact, this is the model from which our analysis departs, for the reasons argued above in section 1.1.

1.2.2 The impact of transitory income shocks on the current account

In this section first we briefly go through the main predictions of the intertemporal approach to the current account. Then the impact of transitory income shocks on the current account is discussed.

After having dedicated much effort during the decade of the 80s specially to build and develop a dynamic-optimizing framework (even though the first models applied to open economies date back to the 1970s), the intertemporal approach to the current account is the main model used by economists today to analyze the impact of real variables on the current account. We refer to Obstfeld and Rogoff (1995, 1996), Razin (1995), and Frenkel, Razin and Yuen (1996) for comprehensive surveys on the approach. The intertemporal approach provided a much-needed *riposte* to the traditional Mundell-Fleming model. Three brief references encapsulate the essence of the intertemporal approach:

- According to Sachs (1981, p. 212), "Current account movements are best analyzed in a dynamic macroeconomic model. This is because current account surpluses or deficits represents national savings or borrowing vis-à-vis the rest of the world and therefore are the outcome of intertemporal choices of households, firms, and governments." Therefore, he adds, "expectations of future events" are fundamental to analyze current accounts.

- According to Razin (1995, p. 169), "the models developed have typically emphasized the effects on the current-account balance of real factors such as productivity, the terms of trade, and government spending and taxes, which operate through intertemporal substitution in consumption, production, and investment", stemming from Fisher's (1930) work and posterior applications to consumption and saving decisions, investment, and so on, whereas

- According to Obstfeld and Rogoff (1995, p. 1732), "the intertemporal approach views the current-account balance as the outcome of forward-looking dynamic saving and investment decisions".

In order to analyze how a transitory income shock affects the current account, we focus on a simple deterministic model of the intertemporal approach, borrowed from Obstfeld and Rogoff (1996, p. 74). According to the intertemporal approach, current account balance in period t, CA_t, is given by

$$CA_t = (Y_t - \widetilde{Y}_t) - (I_t - \widetilde{I}_t) - (G_t - \widetilde{G}_t), \tag{1.1}$$

when the subjective discount rate is equal to the constant world discount factor $1/(1+r)$ and there are no borrowing or lending constraints. The variables Y_t, I_t, and G_t denote the level of output, investment and government spending in period t, respectively. The variables $\widetilde{Y}_t$, $\widetilde{I}_t$, and $\widetilde{G}_t$ denote the permanent level of output, investment and government spending, respectively, where

$$\widetilde{X}_t \equiv \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} X_s.$$

This means that the permanent level of a variable is the annuity value at the actual interest rate. Then we see that, according to the intertemporal approach to the current account, output values above its permanent value generate surpluses in the current account (i.e. the country accumulates foreign assets that produce interest) due to the smoothing of consumption. Investment values above its permanent values tend to produce deficits in the current account. That means new investment projects are financed borrowing from abroad instead of using domestic savings only. Similarly government spending above its permanent value produces deficits in the current account since the impact of the shock is smoothed over time. As Obstfeld and Rogoff (1996, p. 75) point out, equation (1.1) “is often used to understand the current account’s response to one-time, unanticipated events that jolt the economy to a new perfect foresight path”. Thus, following Kraay and Ventura (KV hereafter)(2000, p. 1138), “in existing intertemporal models of the current account, countries invest the marginal unit of wealth in foreign assets. As a result, these models predict that favorable transitory income shocks generate current accounts responses that are equal to the saving generated by the shock”, so that “all countries respond to transitory income shocks with surpluses in the current account”. KV term it “the traditional rule”.

From an empirical perspective, two methodologies have been used to test the intertemporal approach to the current account. The first methodology stems from Hall's (1978) work to apply the assumption of rational expectations to the consumption theory based on forward looking expectations. It is based on the test of a more or less sophisticated version of equation (1.1). The second methodology, pioneered by Feldstein and Horioka (1980), is based on the relation existing between national saving and investment rates. The second one is the approach we follow here. Feldstein and Horioka (1980, p. 317) wanted to "[...] measure the extent to which a higher domestic saving rate in a country is associated with a higher rate of domestic investment.", so that "with perfect world capital mobility, there should be no relation between domestic saving and domestic investment: saving in each country responds to the worldwide opportunities for investment while investment in that country is financed by the worldwide pool of capital." They find that the empirical evidence runs in favor of a strong relationship between both variables, thus attributing it to the lack of perfect world capital mobility. According to Frankel (1992, p. 41), "Feldstein and Horioka upset conventional wisdom in 1980 when they concluded that changes in countries' rate of national saving has very large effects on their rates of investment and interpreted this finding as evidence of low capital mobility". The paradox of having perfect capital mobility going along with a strong association between savings and investment has been termed the "Feldstein-Horioka puzzle".

Today many economists do not share Feldstein and Horioka's conclusion: they attribute the important relationship between saving and investment to the existence of significant common sources of variation in saving and investment. However, "it seems likely that of many potential explanations of the Feldstein-Horioka results, no single one fully explains the behavior of all countries", according to Obstfeld and Rogoff (1995, p. 1779). In fact, as Ventura (2003, p. 495) puts it, "But two decades and hundreds (thousands?) of regressions after Feldstein and Horioka (1980), I am quite sceptical that we will ever find these common sources of variation." We should note that Feldstein and Horioka (1980, p. 319) were aware that a high association "could reflect other common causes of the variation in both saving and investment", but they argue that a high association "would however be strong evidence against the hypothesis of perfect capital mobility and would place on the defenders of that hypothesis the burden of identifying such common causal factors." The actual situation is perfectly summarized by Obstfeld and Rogoff (2000, p. 339): "International macroeconomics is a field replete with truly perplexing puzzles, and we generally have five to ten (or more) alternative answers to each of them. These answers are typically very clever but very far from thoroughly convincing, and so the puzzles remain. [...] Why do observed OECD current-account imbalances tend to be so small relative to saving and investment when measured over any sustained period (the Feldstein-Horioka puzzle)?". Therefore, the finding of Feldstein and Horioka (1980) is still one of the "six major puzzles in international macroeconomics" [borrowed from the title of the paper by Obstfeld and Rogoff (2000)], more than twenty years later, even though recent empirical studies suggest that the Feldstein-Horioka finding seems to be losing some support in the euro area (Blanchard and Giavazzi, 2002).

KV (2000) have recently offered a brilliant departure from the standard model to the intertemporal approach, that is, the traditional rule. Based on a stochastic small open economy model in continuous time, KV (2000, p. 1138) show that if, instead of assuming that in the face of transitory income shocks countries invest all the amount saved in foreign assets, we assume that "the country invests the marginal unit of wealth as the average one" then we obtain that "the current account response is equal to the saving generated by the shock multiplied by the country's share of foreign assets in total assets" (p. 1137). They term it "the new rule". Thus the net foreign asset position of the country, either creditor or debtor, is the key variable around which hinges the impact of a transitory income shock on the current account. The empirical evidence for thirteen OECD countries for the 1973-1995 period seems to support the new rule. Therefore, KV provide a new framework that coherently relates the theory on the intertemporal approach to the current account and the evidence on current accounts.

1.3 Structure of the thesis

Three essays, Chapters 2, 3 and 4, are the backbone of the thesis. The structure of the thesis is as follows. Chapter 2 (Essay 1) studies the impact of the expenditure policy of the public sector and risk on the world economy, assuming that the spending of the public sector is utility-enhancing. Once the world equilibrium is characterized, we study the impact of changes in exogenous variables (public sector, risk, and so on) on key economic variables such as consumption-wealth ratio, the growth rate of wealth and welfare, provided that the size of the public sector is exogenously given. A higher weight of public consumption on the utility function raises the growth rate due to a fall in consumption-wealth ratio. Then we derive that open economies should have higher consumption-wealth ratio and welfare, and we discuss whether open economies should have higher growth rates or not. Next, the optimal size of the public sector is derived and we show the impact of changes in exogenous variable on this size. For example, a higher covariance between domestic and foreign productivity shocks raises the optimal size of the public sector. Then we obtain that open economies should have a higher size of the public sector than closed economies, under more general conditions that those established in Turnovsky (1999).

In Chapter 3 (Essay 2) we pursue the same objective of Chapter 2, but instead we postulate that public spending is productivity- and volatility-enhancing. Assuming that the size of the public sector is exogenous, we obtain the same conclusions than those in Chapter 2. In addition, we obtain the optimal size of the public sector. Then we compare the welfare maximizing size with the growth maximizing size. We discuss whether more open economies should have a higher size of the public sector. Focusing on the case that public spending is productive only, we find that the optimal size in an open economy is higher than that in a closed economy if and only if the optimal size in a foreign closed economy is higher than that in a domestic closed economy. In addition, in the more general case that public spending is volatility-enhancing besides productivity-enhancing, risk diversification plays a crucial role explaining why open economies should have higher optimal size of the public sector.

Chapter 4 (Essay 3) analyzes the impact of transitory income shocks on the current account, extending the new rule suggested by KV to a two-country world. After reviewing the traditional rule and the new rule, we find that, according to the extended new rule, the impact of a transitory income shock depends on the difference between the rates of growth of both economies, in addition to the debtor or creditor position of the country posited by the new rule. Then we empirically test the traditional rule, the new rule and the extended new rule with the same sample used by KV, so that results can be more easily compared. We find that the extended new rule provides important additional insights to the new rule to account for the empirical evidence. However, the empirical validation of the extended new rule is far from being conclusive.

In Chapter 5 the main results are summarized, and we suggest possible avenues for future research.

Finally, I should point out that the three essays of this thesis have been written in a self-contained way, so that each of them can be independently read. As a result of that, the reader of this thesis will surely find some passages of the thesis repetitive. I hope that the reader will be benevolent with the repetitions.

Chapter 2

Risk, utility-enhancing government expenditure, and the world economy

2.1 Introduction

The role of government expenditure policy in the long run behavior of the economy has received considerable attention in recent years, specially due to the advent of endogenous growth models. That is not surprising since in the Solow-Swan neoclassical growth model "conventional macroeconomic policy had no influence on long-run growth performance" (Turnovsky, 2003, p. 1). Barro (1990) pioneered the analysis based on a closed economy deterministic AK growth model where public spending influences utility.[1] This has led to others, Turnovsky (1996, 1999) for example, to incorporate small open economy features and risk into endogenous growth models where public spending is utility-enhancing. Thus, substantial conclusions have been derived regarding the impact of risk and the expenditure policy of the public sector on the economy, and the optimal size of the public sector, provided that the spending of the public sector enhances utility. However, analysis based on two-country stochastic models are badly needed, specially when financial markets are becoming increasingly integrated.

This paper analyzes the influence of risk and the expenditure policy of the public sector by incorporating utility-enhancing public spending [see

[1] Barro (1990) also analyzed the role of productivity-enhancing government expenditure.

Barro (1990)] into a two-country stochastic AK growth model developed by Turnovsky (1997, Ch. 11). Then the size of the public sector that maximizes welfare can be endogenously derived, instead of exogenously given as in Turnovsky (1997, Ch. 11). Previous papers introduced risk into endogenous growth models, but public spending was neither utility-enhancing nor productive [see, for example, Eaton (1981)]. Turnovsky (1996) extended Barro's (1990) closed economy model by incorporating utility-enhancing government expenditure into a small open economy. Turnovsky (1999) added risk to a small open economy. Therefore, our model has been built up combining the main characteristics of the core literature[2]:

- It is an AK growth model, as the rest of the models.
- It is a two-country model, following the framework set out by Turnovsky (1997, chap. 11), whereas the rest are one-country models (either a closed economy or a small open economy).
- Public consumption is utility enhancing, following the original work by Barro (1990). Thus, the model will be able to determine the size of the public sector that maximizes the welfare of the representative agent, as most of the models in the core literature do. Turnovsky (1997) is the only model that cannot analyze the magnitude of such a size, since public spending is neither utility enhancing nor productive, so that "it can be interpreted as being a real drain on the economy or, alternatively, as some public good that does not affect the marginal utility of private consumption or the productivity of private capital" (Turnovsky, 1997, p. 338). Turnovsky (1996, 1999) extends Barro´s (1990) model from a closed economy to a small open economy setting.
- The model is stochastic. The only models in the core literature that are not stochastic are Barro (1990) and Turnovsky (1996). In this respect, Turnovsky (1999) extends the deterministic models in Barro (1990) and Turnovsky (1996) to a stochastic setting.

Table 2.1 encapsulates the relationship between the model in this paper and the core literature.

[2] As "core literature" we understand the model developed by Turnovsky (1997, Ch. 11) and those papers that have analyzed the impact of risk and/or the expenditure policy of the public sector on long-term growth based on AK growth models, provided that public spending is utility-enhancing.

Table 2.1: An overview of the model

Existing models	AK growth	Two countries	Size of the public sector	Stochastic shocks
Barro (1990)	X		X	
Turnovsky (1996)	X		X	
Turnovsky (1997, chap. 11)	X	X		X
Turnovsky (1999)	X		X	X
This model	X	X	X	X

We believe that this model can be specially useful at the present moment of the European Economic and Monetary Union (EMU). First, countries within the euro area have adopted the Stability and Growth Pact (SGP) from 1st January 1999 onwards. The objective of the SGP is that countries within the euro area attain budget balance, in the medium or in the long run, so that the assumption of continuous budget balance that we make in this paper seems reasonable. Second, the emphasis of this paper is on the long run and, therefore, it does not focus on the influence of business cycles, important as they may be. Third, there exists a recurrent preoccupation regarding whether the shocks that affect European countries are becoming more idiosyncratic (asymmetric) or not, and the consequences of such a pattern. In this paper we shall pay special attention to the influence that a change in the correlation between domestic and foreign productivity shocks, and public spending shocks generate on the world economy, whereas the core literature has not analyzed such an issue. Fourth, there is a permanent debate about whether the size of the public sector should be bigger or smaller and, more specifically, whether more open economies should have bigger governments or not. Rodrik (1998) showed that economies that are more open to international trade have bigger governments and argues that it is due to the fact that government spending provides social insurance against external risk. However, Alesina and Wacziarg (1998) show that the link between the size of the public sector and openness can be explained alternatively on the grounds that a higher size of the public sector is related to small economies (due to the economies of scale involved in the provision of public goods) and that small economies are usually more open to trade. Then country size is the variable that can account for the positive relation between the size of the public sector and the openness to trade. The model in this paper sheds some light on the issue, since it compares the size of the public sector that maximizes the welfare in an open economy to that in a closed economy.

We start by analyzing the impact of risk and the public sector on consumption-wealth ratio, the growth rate of assets, and welfare, once the macroeconomic equilibrium has been characterized. Then, the results of an open economy in contrast to those of a closed economy are compared. Next, the welfare-maximizing size of the public sector is derived. We discuss whether maximizing growth is equivalent to maximizing welfare, and we analyze the impact of exogenous parameters, risk specially, on the optimal size. Whether more open economies should have a higher size of the public sector is discussed. Finally, we conclude by indicating possible avenues for future research.

2.2 The world economy

2.2.1 Basic structure

The world economy consists of two countries, each of them producing only one homogeneous good. On each country exist a representative agent and a public sector, both with an infinite time horizon. This economy is a real one, that is, there are no nominal assets, such as money, different financial assets, etc. Unstarred variables refer to domestic economy, whereas starred variables refer to foreign economy. The development of this model will focus on the domestic economy given that the results for the foreign economy are very similar.

The homogeneous good produced by both countries can be either consumed or invested in capital without having to incur in any kind of adjustment costs. We assume that domestic production can be obtained using only domestic capital, K, through an AK function, and that it can be expressed through a first order stochastic differential equation, so that production flow dY (the variation of the state variable) is not completely determined, but subject to a stochastic disturbance

$$dY = \alpha K dt + \alpha K dy,$$

where $\alpha > 0$ is the (constant) marginal physical product of capital and dy represents a proportional domestic productivity shock. More precisely, dy

is the increment of a stochastic process y. Those increments are temporally independent and are normally distributed, and they satisfy that $E(dy) = 0$ and $E(dy^2) = \sigma_y^2 dt$.[3] We omit, for convenience, formal references to time, although those variables depend on time. We must note that dY indicates the flow of production, instead of Y, as is ordinarily done in stochastic calculus.

The foreign economy is structured symmetrically to the domestic economy. Thus, foreign production is carried out using capital domiciled abroad, K^*, with a production function very similar to the one in the domestic economy

$$dY^* = \alpha^* K^* dt + \alpha^* K^* dy^*,$$

where $\alpha^* > 0$ is the marginal physical product of capital and dy^* represents a proportional foreign productivity shock. More precisely, dy^* is the increment of a stochastic process y^*. Those increments are temporally independent and are distributed normally, satisfying that $E(dy^*) = 0$ and that $E(dy^{*^2}) = \sigma_{y^*}^2 dt$.

Both domestic capital, K, and foreign capital, K^*, can be owned by the domestic representative agent or the foreign representative agent. The subscript d denotes the holdings of assets of the domestic representative agent and the subscript f denotes the holdings of assets of the foreign representative agent. So it must be satisfied that

$$\begin{aligned} K &= K_d + K_f \\ K^* &= K_d^* + K_f^*. \end{aligned}$$

The wealth of the domestic representative agent, W, and the wealth of the foreign representative agent, W^*, therefore will be

$$W = K_d + K_d^* \tag{2.1}$$
$$W^* = K_f + K_f^*. \tag{2.2}$$

[3] That is, the production flow follows a Brownian motion with drift αK and with variance $\alpha^2 K^2 \sigma_y^2$.

2.2.2 Domestic economy

The maximization problem

The preferences of the domestic representative agent are represented by a constant elasticity of substitution (or isoelastic) intertemporal utility function where she obtains utility from private consumption, C, and from public consumption, G

$$E_0 \int_0^{\infty} \frac{1}{\gamma}(CG^{\eta})^{\gamma} e^{-\beta t} dt \tag{2.3}$$
$$-\infty < \gamma < 1; \eta > 0; \gamma\eta < 1; \gamma(1+\eta) < 1.$$

The welfare of the domestic representative agent in period 0 is the expected value of the discounted sum of instantaneous utilities, conditioned on the set of disposable information in period 0. The parameter β is a positive subjective discount rate (or rate of time preference). For the isoelastic utility function the Arrow-Pratt coefficient of relative risk aversion is given by the expression $1-\gamma$. When $\gamma = 0$ this function corresponds to the logarithmic utility function. The empirical evidence suggests a high degree of relative risk aversion, so that $\gamma < 0$ (Campbell, 1996). The parameter η measures the influence of public consumption on the welfare of the domestic representative agent. We assume that both private consumption and public consumption generate a positive marginal utility, so that $\eta > 0$. The other restrictions on the utility function are necessary to ensure concavity with respect to private consumption and public consumption.

The domestic representative agent consumes at a deterministic rate $C(t)dt$ in the instant dt and must pay the corresponding taxes and thus the dynamic budget restriction can be expressed in the following way

$$dW = [\alpha K_d + \alpha^* K_d^*]\, dt + [\alpha K_d dy + \alpha^* K_d^* dy^*] - C dt - dT, \tag{2.4}$$

where dT denotes the taxes the domestic representative agent must pay to the public sector. The structure of taxes will be detailed below.

There is a public sector besides the domestic representative agent. Public sector spending, dG, increases with wealth, so we can achieve a balanced growth path[4]. Public spending evolves according to

[4]Other rules can also achieve a balanced growth path. See Turnovsky (1996) for more details.

$$dG = gWdt + Wdz, \tag{2.5}$$

where $g = G/W$ is the size of the public sector and dz is the increment of a stochastic process z. Those increments are temporally independent and are normally distributed, satisfying that $E(dz) = 0$ and $E(dz^2) = \sigma_z^2 dt$. Public sector spending is financed solely via tax collection: the public sector equilibrates its budget continuously, which seems reasonable in the long run, as is the focus of this paper. Therefore, public deficits are not allowed, that is,

$$dT = dG. \tag{2.6}$$

Combining equations (2.5) and (2.6), and plugging them into (2.4), we get the following restriction for the resources of the domestic economy

$$dW = [\alpha K_d + \alpha^* K_d^* - C - gW]\, dt + [\alpha K_d dy + \alpha^* K_d^* dy^* - W dz]\,. \tag{2.7}$$

Let us remember that the holding of assets by the domestic representative agent is subject to the domestic wealth equation (2.1). If we define the following variables for the domestic representative agent

$$\begin{aligned} n_d &\equiv \frac{K_d}{W} = \text{ share of the domestic portfolio materialized} \\ &\text{in domestic capital} \\ n_d^* &\equiv \frac{K_d^*}{W} = \text{ share of the domestic portfolio materialized} \\ &\text{in foreign capital,} \end{aligned}$$

equation (2.1) can be expressed more conveniently as

$$1 = n_d + n_d^* \tag{2.8}$$

and substituting those variables into the budget constraint (2.7) we obtain the following dynamic restriction for the resources of the domestic economy

$$\frac{dW}{W} = \left[\alpha n_d + \alpha^* n_d^* - \frac{C}{W} - g\right] dt + \left[\alpha n_d dy + \alpha^* n_d^* dy^* - dz\right]. \tag{2.9}$$

This equation can be more conveniently expressed as

$$\frac{dW}{W} = \psi dt + dw, \tag{2.10}$$

where the deterministic and stochastic parts of the rate of accumulation of assets, dW/W, can be expressed in the following way

$$\psi \equiv n_d \left[\alpha - \alpha^*\right] + \alpha^* - g - \frac{C}{W} \equiv \rho - g - \frac{C}{W} \tag{2.11}$$

$$dw \equiv n_d \left[\alpha dy - \alpha^* dy^*\right] + \alpha^* dy^* - dz, \tag{2.12}$$

where $\rho \equiv \alpha n_d + \alpha^* n_d^* \equiv n_d \left[\alpha - \alpha^*\right] + \alpha^*$ denotes the gross rate of return of the asset portfolio.

Equilibrium

The objective of the domestic representative agent consists in choosing the path of private consumption and portfolio shares that maximize the expected value of the intertemporal utility function (2.3), subject to $W(0) = W_0$, (2.10), (2.11), and (2.12). This optimization is a stochastic optimum control problem.[5] Initially we assume that the government establishes an arbitrarily exogenous size of the public sector, g. We analyze the case in which such a size is chosen optimally in section 2.4.

It is important to bear in mind that the domestic agent takes as given the rates of return of different assets, as well as the corresponding variances and covariances. However, these parameters will endogenously be determined in the macroeconomic equilibrium we shall obtain.

The first step in order to solve this optimization problem is to introduce a value function, $V(W)$, which is defined as

[5] To solve problems of stochastic optimum control see, for example, Kamien and Schwartz (1991, section 22), Malliaris and Brock (1982, ch. 2), Obstfeld (1992), or Turnovsky (1997, ch. 9; 2000, ch. 15).

$$V(W) = Max_{\{C,n_d\}} \ E_0 \int_0^\infty \frac{1}{\gamma}(CG^\eta)^\gamma e^{-\beta t} dt, \tag{2.13}$$

subject to restrictions (2.10), (2.11), and (2.12) and given initial wealth. The value function in period 0 is the expected value of the discounted sum of instantaneous utilities, evaluated along the optimal path, starting in period 0 in the state $W(0) = W_0$.

Second, starting from equation (2.13) the value function must satisfy the following equation, known as the Hamilton-Jacobi-Bellman equation of stochastic control theory or, for short, the Bellman equation

$$\beta V(W) = Max_{\{C,n_d\}} \ \left[\frac{1}{\gamma}(CG^\eta)^\gamma + V'(W)W\psi + 0.5V''(W)W^2\sigma_w^2\right]. \tag{2.14}$$

Third, (2.14) is partially differentiated with respect to C and n_d in order to get the first order optimality conditions of this problem

$$\begin{aligned} C^{\gamma-1}G^{\eta\gamma} - V'(W) &= 0 \quad (2.15) \\ V'(W)W(\alpha - \alpha^*) + V''(W)W^2 cov\left[dw, \alpha dy - \alpha^* dy^*\right] &= 0. \quad (2.16) \end{aligned}$$

The solution to this maximization problem is obtained through trial and error. We seek to find a value function $V(W)$ that satisfies, on the one hand, the first order optimality conditions and, on the other, the Bellman equation. In the case of isoelastic utility functions the value function has the same form of the utility function [Merton (1969), generalized in Merton (1971)]. Thus, we guess that the value function is of the form

$$V(W) = AW^{\gamma(1+\eta)}, \tag{2.17}$$

where the coefficient A is determined below. That guess implies

$$\begin{aligned} V'(W) &= A\gamma(1+\eta)W^{\gamma(1+\eta)-1} \\ V''(W) &= A\gamma(1+\eta)\left[\gamma(1+\eta) - 1\right]W^{\gamma(1+\eta)-2}. \end{aligned}$$

Inserting these expressions into the first order optimality conditions (2.15) and (2.16), the result is

$$\begin{aligned} C^{\gamma-1}G^{\eta\gamma} &= A\gamma(1+\eta)W^{\gamma(1+\eta)-1} && (2.18)\\ (\alpha-\alpha^*)\,dt &= [1-\gamma(1+\eta)]\,cov\,[dw, \alpha dy-\alpha^* dy^*]\,. && (2.19) \end{aligned}$$

Both are typical equations in stochastic models in continuous time. Equation (2.18) indicates that at the optimum, the marginal utility derived from private consumption must be equal to the marginal change in the value function or the marginal utility of wealth. Equation (2.19) shows that the optimal choice of portfolio shares of the domestic representative agent must be such that the risk-adjusted rates of return of both domestic and foreign capital are equalized.

Combining (2.18) and (2.19), and substituting them in the equation (2.14), we are able to calculate, after some algebra, the equilibrium portfolio shares and the consumption-wealth ratio in the domestic open economy

$$\begin{aligned} n_d &= \frac{\alpha-\alpha^*}{[1-\gamma(1+\eta)]\,\Delta}\\ &\quad+\frac{\alpha^{*^2}\sigma_{y^*}^2-\alpha\alpha^*\sigma_{yy^*}+\alpha\sigma_{yz}-\alpha^*\sigma_{y^*z}}{\Delta} && (2.20)\\ n_d^* &= 1-n_d && (2.21)\\ \left(\frac{C}{W}\right)_o &= \frac{1}{(1-\gamma)(1+\eta)}\,[\beta-\gamma(1+\eta)\,(\rho-g)\\ &\quad+0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma_{w,o}^2]\,, && (2.22) \end{aligned}$$

where

$$\begin{aligned} \Delta &= \alpha^2\sigma_y^2-2\alpha\alpha^*\sigma_{yy^*}+\alpha^{*^2}\sigma_{y^*}^2 && (2.23)\\ \sigma_{w,o}^2 &= n_d^2\alpha^2\sigma_y^2+2n_dn_d^*\alpha\alpha^*\sigma_{yy^*}+n_d^{*^2}\alpha^{*^2}\sigma_{y^*}^2+\sigma_z^2\\ &\quad-2n_d\alpha\sigma_{yz}-2n_d^*\alpha^*\sigma_{y^*z}. && (2.24) \end{aligned}$$

Do note that neither the expression Δ nor the variance of the rate of accumulation of domestic assets, $\sigma_{w,o}^2$, can be negative and the variables with the subscript o refer to values in an open economy.

Then, the equilibrium rate of wealth accumulation of the open domestic economy follows the stochastic process

$$\frac{dW}{W} = \psi_o dt + dw_o, \tag{2.25}$$

where the deterministic and stochastic components are, respectively

$$\begin{aligned}
\psi_o &= \frac{1}{(1-\gamma)(1+\eta)} \{(1+\eta)(\rho - g) - \beta \\
&\quad -0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma^2_{w,o}\} &(2.26)\\
dw_o &= n_d \alpha dy + n_d^* \alpha^* dy^* - dz. &(2.27)
\end{aligned}$$

Even though with more general utility functions, portfolio shares and consumption-wealth ratio will be functions of time, in this model all those variables are constant because the utility function exhibits constant relative risk aversion, the production function is linear, and the mean and variances of the underlying stochastic processes are stationary: the equilibrium is characterized by balanced real growth, where all the (real) assets grow at the same rate, and by constant consumption-wealth ratio and portfolio shares. In addition, we should observe that portfolio shares do not depend on the size of the public sector, but they do depend on the degree of relative risk aversion. The result is very similar to Turnovsky (1997, ch. 11). However, we should note that portfolio shares also depend on the parameter that reflects the influence of public consumption in the utility function of the domestic representative agent, η. The same is also true for the foreign economy, as we shall see below.

Now we describe the behavior of the domestic economy if it were closed in order to compare the results of an open economy with those of a closed economy later on. In a model of perfect capital mobility such as this, where domestic and foreign assets are traded without restrictions, we use the shares of the domestic portfolio materialized in domestic and foreign capital, n_d and n_d^* respectively, to approximate the degree of openness of the domestic economy. Since our emphasis is on the trade of assets, then we call closed economy the situation where there is no trade of assets. However, we should bear in mind that what we call closed economy is compatible with positive amounts of exports and imports, but subject to the restriction that the trade of goods must be balanced. In the case of a closed economy, the equilibrium solution will be given by the expressions

$$
\begin{aligned}
\left(\frac{C}{W}\right)_c &= \frac{1}{(1-\gamma)(1+\eta)}\{\beta - \gamma(1+\eta)(\alpha - g) \\
&\quad +0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma^2_{w,c}\} && (2.28) \\
\sigma^2_{w,c} &= \alpha^2\sigma^2_y + \sigma^2_z - 2\alpha\sigma_{yz} && (2.29) \\
\psi_c &= \frac{1}{(1-\gamma)(1+\eta)}\{(1+\eta)(\alpha - g) - \beta \\
&\quad -0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma^2_{w,c}\} && (2.30) \\
dw_c &= \alpha dy - dz,
\end{aligned}
$$

where the variables with the subscript c refer to values in a closed economy.

In order to guarantee that consumption is positive in the domestic open economy we impose the feasibility condition that the marginal propensity to consume out of wealth must be positive since wealth does not become negative

$$
\begin{aligned}
&\frac{1}{(1-\gamma)(1+\eta)}\{\beta - \gamma(1+\eta)(\rho - g) \\
&+0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma^2_{w,o}\} > 0.
\end{aligned}
$$

For the first order optimality conditions to characterize a maximum, the corresponding second order condition must be satisfied, that is, the Hessian matrix associated to the maximization problem and evaluated at the optimal values of the choice variables

$$
\begin{bmatrix}
(\gamma-1)(V'(W))^{\frac{\gamma-2}{\gamma-1}} & 0 \\
0 & V''(W)W^2\Delta
\end{bmatrix}
$$

must be negative definite,[6] which implies that

[6] See Chiang (1984, pp. 320-323), for example.

$$\begin{aligned}(\gamma-1)\left(V'(W)\right)^{\frac{\gamma-2}{\gamma-1}} &< 0 \\ V''(W)W^2\Delta &< 0,\end{aligned}$$

where $\Delta > 0$ (in a risky economy) was already defined in equation (2.23). To evaluate those conditions, first we obtain the value of the coefficient A in equation (2.18)

$$A = \frac{g^{\eta\gamma}}{\gamma(1+\eta)}\left(\frac{C}{W}\right)^{\gamma-1}, \tag{2.31}$$

where C/W is the optimal value pointed out by equation (2.22). Then we insert (2.31) into the value function (2.17). Noting that $g = G/W$, the value function is given, after some algebra, by

$$V(W) = \frac{g^{\eta\gamma}}{\gamma(1+\eta)}\left(\frac{C}{W}\right)^{\gamma-1} W^{\gamma(1+\eta)}, \tag{2.32}$$

where we can observe that, given the restrictions on the utility function, $V'(W) > 0$ and $V''(W) < 0$ provided that $C/W > 0$.

In addition, we impose that the macroeconomic equilibrium must satisfy the transversality condition so as to guarantee the convergence of the value function

$$\lim_{t\to\infty} E\left[V(W)\, e^{-\beta t}\right] = 0. \tag{2.33}$$

Now let us show that should the feasibility condition be satisfied, that would be equivalent to satisfy the transversality condition.[7] To evaluate (2.33), we start expressing the dynamics of the accumulation of wealth

$$dW = \psi W dt + W dw. \tag{2.34}$$

The solution to equation (2.34), starting from the initial wealth $W(0)$, is[8]

[7] See Merton (1969). Turnovsky (2000) provides, for example, the proof of the transversality condition as well.

[8] See Malliaris and Brock (1982, pp. 135-136), for example.

$$W(t) = W(0)e^{(\psi - 0.5\sigma_w^2)t + w(t) - w(0)}.$$

Since the increments of w are temporally independent and are normally distributed then[9]

$$\begin{aligned} E[AW^{\gamma(1+\eta)}e^{-\beta t}] &= E[AW(0)^{\gamma(1+\eta)}e^{\gamma(1+\eta)(\psi-0.5\sigma_w^2)t+\gamma(1+\eta)[w(t)-w(0)]-\beta t}] \\ &= AW(0)^{\gamma(1+\eta)}e^{\left[\gamma(1+\eta)(\psi-0.5\sigma_w^2)+0.5\gamma^2(1+\eta)^2\sigma_w^2-\beta\right]t}. \end{aligned}$$

The transversality condition (2.33) will be satisfied if and only if

$$\gamma(1+\eta)\left\{\psi - 0.5\gamma(1+\eta)\left[1-\gamma(1+\eta)\right]\sigma_w^2\right\} - \beta < 0.$$

Now substituting equations (2.11) and (2.22), it can be shown that this condition is equivalent to

$$\frac{C}{W} > 0,$$

and thus feasibility guarantees convergence as well.

Finally, it should be noted that since the public sector equilibrates its budget continuously, the intertemporal budget constraint of the public sector is satisfied trivially.

2.2.3 Foreign economy

The maximization problem

The problem facing the foreign representative agent can be formulated in an analogous way. Her preferences are represented by the following intertemporal utility function

$$\begin{aligned} & E_0 \int_0^\infty \frac{1}{\gamma^*}(C^* G^{*\eta^*})^{\gamma^*} e^{-\beta^* t} dt \\ -\infty &< \gamma^* < 1; \eta^* > 0; \gamma^*\eta^* < 1; \gamma^*(1+\eta^*) < 1. \end{aligned}$$

[9] See Malliaris and Brock (1982, pp. 137-138), for example.

The equation of the rate of accumulation of wealth of the foreign representative agent can be expressed as

$$\frac{dW^*}{W^*} = \psi^* dt + dw^*,$$

where

$$\begin{aligned} \psi^* &\equiv n_f\alpha + n_f^*\alpha^* - g^* - \frac{C^*}{W^*} \equiv \rho^* - g^* - \frac{C^*}{W^*} \\ dw^* &\equiv n_f\alpha dy + n_f^*\alpha^* dy^* - dz^*. \end{aligned}$$

Equilibrium

The equilibrium portfolio shares and consumption-wealth ratio in the foreign economy are

$$\begin{aligned} n_f &= \frac{\alpha - \alpha^*}{[1-\gamma^*(1+\eta^*)]\,\Delta} + \frac{{\alpha^*}^2\sigma_{y^*}^2 - \alpha\alpha^*\sigma_{yy^*} + \alpha\sigma_{yz^*} - \alpha^*\sigma_{y^*z^*}}{\Delta} \\ n_f^* &= 1 - n_f \\ \left(\frac{C^*}{W^*}\right)_o &= \frac{1}{(1-\gamma^*)(1+\eta^*)}\left\{\beta^* - \gamma^*(1+\eta^*)(\rho^* - g^*)\right. \\ &\quad \left. -0.5\gamma^*(1+\eta^*)\left[\gamma^*(1+\eta^*) - 1\right]\sigma_{w^*,o}^2\right\}, \end{aligned}$$

where

$$\begin{aligned} \sigma_{w^*,o}^2 &= n_f^2\alpha^2\sigma_y^2 + 2n_f n_f^*\alpha\alpha^*\sigma_{yy^*} + {n_f^*}^2{\alpha^*}^2\sigma_{y^*}^2 \\ &\quad +\sigma_{z^*}^2 - 2n_f\alpha\sigma_{yz^*} - 2n_f^*\alpha^*\sigma_{y^*z^*}. \end{aligned}$$

The equilibrium rate of accumulation of wealth in the foreign economy follows the stochastic process

$$\frac{dW^*}{W^*} = \psi_o^* dt + dw_o^*$$

where its deterministic and stochastic components are, respectively

$$\begin{aligned}\psi_o^* &= \frac{1}{(1-\gamma^*)(1+\eta^*)}\left\{(1+\eta^*)(\rho^*-g^*)-\beta^*\right. \\ &\left.-0.5\gamma^*(1+\eta^*)\left[\gamma^*(1+\eta^*)-1\right]\sigma_{w^*,o}^2\right\} \\ dw_o^* &= n_f\alpha dy + n_f^*\alpha^* dy^* - dz^*.\end{aligned}$$

2.3 Equilibrium analysis

In this section first we first review the impact of changes in exogenous variables on the consumption-wealth ratio, the growth rate of wealth of the domestic economy, and welfare, given that most of the results are standard[10]. Then, the results of an open economy are compared to those of a closed economy.

2.3.1 Consumption

The optimal consumption-wealth ratio shown in equation (2.22) is standard in the literature[11]: the consumption function is a linear function of wealth. First, we review how consumption responds to changes in exogenous variables that are not directly related to risk or to the influence of the public sector. Thus, a higher subjective discount rate, β, increases consumption-wealth ratio, because the domestic representative agent finds more attractive to dedicate a higher proportion of wealth to consumption, thus reducing investment. In addition, a higher gross rate of return of the asset portfolio, ρ, raises (reduces) consumption-wealth ratio if $\gamma < (>) 0$ and does not change if $\gamma = 0$. That is the overall result of two opposite effects, substitution and income effects. A higher gross rate of return of the asset portfolio has always a negative substitution effect since consumption becomes less attractive whereas investment is more attractive. The income effect on the consumption-wealth

[10] We refer to Turnovsky (1997, Ch. 11) for the analysis of the impact of production risk and public spending on portfolio shares and on the variance of the growth rate of the domestic economy.

[11] See Merton (1969) for the pioneer work in continuous time with uncertainty. We refer to Turnovsky (1996; 1997, Ch. 11; 1999) for more details on the impact of changes in exogenous variables on consumption-wealth ratio.

ratio originated by a higher gross rate of return of the asset portfolio, is equal to unity: it makes possible to raise both actual and future consumption. If $\gamma < (>)0$, income (substitution) effect dominates substitution (income) effect and if $\gamma = 0$ the two effects compensate each other. From here onwards whenever a result depends on the sign of the parameter γ, we shall only focus on the case where $\gamma < 0$, for being the most empirically relevant situation.

Second, we study the influence of variables related to risk, but not affected by the behavior of the public sector. Thus, the effect of a higher coefficient of risk aversion, γ, on consumption is ambiguous. In addition, a higher variance of the growth rate, $\sigma_{w,o}^2$, reduces consumption-wealth ratio if $\gamma < 0$. Substitution and income effects arise again: totally differentiating equation (2.22) we can easily show that an increase on the variance of the growth rate is equivalent to a fall in the gross rate of return of the asset portfolio, ρ, of $0.5\left[1-\gamma(1+\eta)\right]$. An analogous conclusion applies to the impact of a higher variance of domestic productivity shocks, σ_y^2, a higher variance of foreign productivity shocks, $\sigma_{y^*}^2$,or a higher covariance between domestic and foreign productivity shocks, σ_{yy^*}, on consumption-wealth ratio.

Third, the role of the public sector is analyzed. Consumption-wealth ratio decreases as the size of the public sector, g, increases, for $\gamma < 0$. An increase in the size of the public sector is equivalent to a fall in the gross rate of return of the asset portfolio of 1. In addition, an increase in the variance of public spending shocks, σ_z^2, diminishes consumption-wealth ratio when $\gamma < 0$. An increase in the variance of public spending shocks is equivalent to a fall in the gross rate of return of the asset portfolio of $0.5\left[1-\gamma(1+\eta)\right]$, since the variance of the growth rate increases. In contrast, if either the covariance between domestic productivity shocks and domestic public spending shocks, σ_{yz}, or the covariance between foreign productivity shocks and domestic public spending shocks, σ_{y^*z}, increase, consumption-wealth ratio increases for $\gamma < 0$. That would be due to a reduction in the variance of the growth rate of the domestic economy.

For the case that the utility function is logarithmic, the consumption function becomes much simpler

$$\frac{C}{W} = \frac{\beta}{1+\eta}, \tag{2.35}$$

already found in Turnovsky (1996, 1999). This implies that a higher weight of public consumption in the utility function, η, reduces unambiguously the consumption-wealth ratio. A higher value of η increases the attractiveness of public consumption *in relation to* private consumption, given the exogenous size of the public sector. In addition, any other variable (risk, for example) does not change consumption-wealth ratio, and the consumption function in an open economy is equal to that in a closed economy.

2.3.2 Growth

The mean growth rate of assets achieved in equilibrium, given by (2.26), is standard in the literature[12]. First, we focus on the impact of variables that do not refer neither to risk nor to the public sector, on the growth rate of assets. Thus, a higher subjective discount rate, β, reduces unambiguously the growth rate, given that dedicating resources to consumption becomes more attractive whereas investment is discouraged. In addition, a higher gross rate of return of the asset portfolio, ρ, increases the growth rate, even though consumption-wealth ratio may rise.

Second, we study the influence of variables related to risk, but not affected by the behavior of the public sector. Thus, a change in the parameter γ generates an ambiguous effect on the growth rate. Departing from $\psi_o = \rho - g - (C/W)_o$, this model shows that an increase in the variance of domestic productivity shocks, σ_y^2, shifting investment towards foreign capital, tends on the one hand, to increase the growth rate if $\alpha^* > \alpha$. On the other hand, the growth-enhancing effect is reinforced when $\gamma < 0$, since consumption-wealth ratio falls due to an increase in σ_y^2 (Turnovsky, 1997, p. 442). Similarly, an increase in the variance of the foreign productivity shocks, $\sigma_{y^*}^2$, making domestic capital more attractive, tends to increase the growth rate if $\alpha > \alpha^*$. Again, the positive effect on the growth rate is strengthened if $\gamma < 0$: consumption-wealth ratio falls due to an increase in $\sigma_{y^*}^2$.

Third, we analyze the impact of the public sector on the growth rate. It is easy to show that a higher size of the public sector, g, reduces unambiguously the growth rate of the economy, even though consumption-wealth ratio may fall. A higher variance of domestic public spending, σ_z^2, increases the growth rate of the economy for $\gamma < 0$, because consumption-wealth ratio falls (Turnovsky, 1997, p. 444). In contrast, the opposite conclusions are obtained when either the covariance of domestic productivity and public

[12] We refer to Turnovsky (1996; 1997, Ch. 11; 1999) again for more details on the impact of changes in exogenous variables on the growth rate of wealth.

spending shocks, σ_{yz}, or the covariance of foreign productivity shocks and public spending shocks, σ_{y^*z}, increases.

Fourth, in the case of a logarithmic utility function the growth rate is given by the expression

$$\psi_o = \rho - g - \frac{\beta}{1+\eta}.$$

Thus, a higher value of the parameter η increases unambiguously the growth rate of assets of the domestic economy. Even though it seems counterintuitive at a first glance, the reason behind is that a higher weight of public consumption reduces consumption-wealth ratio, as we saw in the previous section, thus increasing the rate of accumulation of assets of the economy, given the exogenous size of the public sector. In addition, the growth rate of domestic wealth does not have to be equal in an open economy compared to a closed economy, as we shall see below in more detail.

Finally, we conclude that most of the results are standard in the literature, even though they must be adjusted to include utility-enhancing public consumption. As stated by Turnovsky (1997, p. 432), "With identical preferences and portfolios, differences in the international growth rates of wealth and therefore of consumption are due entirely to differences in the respective size of government, $g - g^*$, in the two economies. If the size of government is uniform, then the equilibrium growth rates, ψ and ψ^*, will be identical". However, to account for differences in the rates of growth the parameter η plays here an important role in the model as well. This implies that, having the representative agents of both economies identical preferences, portfolios and sizes of the public sector, differences in the growth rates of both economies can be explained in terms of differences in the weight of public consumption in the utility functions of both economies. We have also shown that economies which assign a higher weight to public consumption in their utility function will have higher growth rates due to lower consumption-wealth ratios.

2.3.3 Welfare

Economic welfare is measured by the value function we have used to solve the problem of intertemporal optimization, given by equation (2.32). From the total differential of equation (2.32) we obtain, after some algebra, that

$$\frac{dV}{V} = (\gamma - 1)\frac{d(C/W)}{C/W} + \gamma\eta\frac{dg}{g}, \tag{2.36}$$

where we can observe that changes in the optimal consumption-wealth ratio and the (exogenous) size of the public sector have an impact on welfare.

First, a higher optimal consumption-wealth ratio can improve or deteriorate the welfare of the domestic representative agent. That is due to the fact that the value function can take either positive or negative values, depending on the sign of the coefficient γ. Since C/W and g are positive in equation (2.32) then $\gamma V(W) > 0$. For the case $\gamma < 0$, anything that increases the optimal consumption-wealth ratio elevates the welfare of the representative agent. Thus, for example, a higher subjective discount rate, increasing the optimal consumption-wealth ratio, generates higher welfare if $\gamma < 0$.

Second, the size of the public sector is an important factor influencing the welfare of the representative agent. Do note that the optimal consumption-wealth ratio, given by equation (2.22), also depends on the size of the public sector, g. Therefore, the impact of changes in the size of the public sector on welfare is given by

$$\frac{dV}{V} = \gamma\left[\eta - \frac{g}{C/W}\right]\frac{dg}{g}.$$

Thus, a higher size of the public sector can increase or reduce the welfare of the domestic representative agent, even though it reduces unambiguously the growth rate. The crucial point lies on whether $g \lesseqgtr \eta C/W$. If $g < \eta C/W$, an increase in the size of the public sector augments the welfare of the representative agent. That is due to the fact that the marginal utility derived from public consumption is higher than the marginal utility derived from private consumption. If $g = \eta C/W$, an increase in the size of the public sector does not alter the welfare of the representative agent because the marginal utility derived from public consumption is equal to the marginal utility derived from private consumption: it is the size of the public sector that maximizes welfare, as we shall see in the next section. Finally, if $g > \eta C/W$, an increase in the size of the public sector reduces the welfare of the representative agent because the marginal utility derived from public consumption is lower than the marginal utility derived from private consumption. These results can be related to the conclusions established in Turnovsky (2000, p. 438): "Thus we infer that increasing the growth rate by reducing government expenditure is not necessarily welfare improving. This will be the case only if initially g is above its optimum". We shall see below that this is completely consistent with the analysis of the size of the public sector that maximizes the welfare of the representative agent.

2.3.4 Open economy versus closed economy

In order to compare the results of an open economy with those of a closed economy, it is convenient to calculate the difference between the variance of the growth rate in an open economy and in a closed economy. Thus if we substract equation (2.29) from equation (2.24) we obtain, after some algebra, that

$$\sigma_{w,o}^2 - \sigma_{w,c}^2 = \Delta n_d^* \left(n_d^* - 2\widetilde{n}_d^* \right), \tag{2.37}$$

where

$$\widetilde{n}_d^* = \frac{\alpha^2 \sigma_y^2 - \alpha\alpha^* \sigma_{yy^*} - \alpha\sigma_{yz} + \alpha^* \sigma_{y^*z}}{\Delta},$$

is the share of the domestic portfolio materialized in foreign capital that minimizes the variance of the growth rate given by equation (2.24).

First, we can compare the consumption-wealth ratio in an open economy to that in a closed economy. If we substract equation (2.28) from equation (2.22) we obtain, using equation (2.37), that, after some algebra,

$$\left(\frac{C}{W}\right)_o - \left(\frac{C}{W}\right)_c = -\frac{1}{1-\gamma}\left\{0.5\gamma\left[1-\gamma\left(1+\eta\right)\right]\Delta n_d^{*^2}\right\}. \tag{2.38}$$

As we can see, the difference between both consumption-wealth ratios depends only on the sign of the parameter γ. Thus, if $\gamma < 0$, then the consumption-wealth ratio will be higher in an open economy than in a closed economy, assuming an interior solution for the value of portfolio shares. An easy way to explain that result can be found focusing on the case $n_d = \widetilde{n}_d$, where

$$\widetilde{n}_d = 1 - \widetilde{n}_d^* = \frac{\alpha^{*^2} \sigma_{y^*}^2 - \alpha\alpha^* \sigma_{yy^*} + \alpha\sigma_{yz} - \alpha^* \sigma_{y^*z}}{\Delta}, \tag{2.39}$$

denotes the share of the domestic portfolio materialized in domestic capital that minimizes the variance of the growth rate of wealth. In such a situation we obtain from equation (2.37) that the variance of the growth rate in an open economy is lower than in a closed economy, $\sigma^2_{w,o} < \sigma^2_{w,c}$. As it was mentioned above, a reduction in the variance of the growth rate is equivalent to an increase in the gross rate of return of the asset portfolio. That, in turn, originates a negative substitution effect and a positive income effect on the consumption-wealth ratio. If $\gamma < 0$ the income effect is stronger than the substitution effect and the consumption-wealth ratio in an open economy is higher than in a closed economy. Additionally, the higher the value of the optimal share of the domestic portfolio materialized in foreign capital, n_d^*, the higher the difference between the results of an open economy with those of a closed economy.

Second, we can compare the growth rate in an open economy to that in a closed economy departing from the equation (2.11) corresponding to an open economy and substracting from it that corresponding to a closed economy

$$\psi_o - \psi_c = n_d^*(\alpha^* - \alpha) - \left[\left(\frac{C}{W}\right)_o - \left(\frac{C}{W}\right)_c\right]. \tag{2.40}$$

We can see that the growth rate in an open economy can be higher than, equal to or lower than that in a closed economy, depending on the signs of the two terms in (2.40). For example, we can establish focusing on the case where $\gamma < 0$, that:

- If $\alpha \geq \alpha^*$, the growth rate in an open economy will be lower than that in a closed economy. The reason behind is that the consumption-wealth ratio in an open economy is higher than that in a closed economy and, additionally, if $\alpha \geq \alpha^*$ the gross rate of return of the asset portfolio in an open economy, ρ, is lower than or equal to the marginal physical product of the domestic capital.

- If $\alpha < \alpha^*$, the growth rate in an open economy can be higher than, equal to or lower than that in a closed economy.

Table 2.2 sums up the comparison between growth rate in an open economy with that in a closed economy given by equation (2.40).

Table 2.2: Comparing growth rates

	$\gamma > 0$	$\gamma = 0$	$\gamma < 0$
$\alpha > \alpha^*$	$\psi_o \lesseqqgtr \psi_c$	$\psi_o < \psi_c$	$\psi_o < \psi_c$
$\alpha = \alpha^*$	$\psi_o > \psi_c$	$\psi_o = \psi_c$	$\psi_o < \psi_c$
$\alpha < \alpha^*$	$\psi_o > \psi_c$	$\psi_o > \psi_c$	$\psi_o \lesseqqgtr \psi_c$

Finally, we can compare the welfare of the domestic representative agent in an open economy to that in a closed economy. As we have shown in equation (2.38), consumption-wealth ratio in an open economy is higher than that in a closed economy for $\gamma < 0$. Going back to the value function given by equation (2.32), we can establish that the welfare of the domestic representative agent is higher in a risky open economy than in a risky closed economy. This result adds some insights to those shown in Obstfeld (1994) and Turnovsky (1997, Ch. 11), where they analyze the impact on welfare when changing from a domestic closed economy with low-yield and no risk (or relatively low risk) assets to an open economy with high-yield and high-risk assets, among other things. Obstfeld (1994, p. 1326-27) showed that "international risk-sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth is the attendant world portfolio shift from safe, but low-yield, capital into riskier, high-yield capital". Additionally, Turnovsky (1997, p. 439) showed that for a logarithmic utility function "the higher growth rate more than offsets the additional risk, and the opportunity to invest in a higher return, higher risk foreign asset improves welfare". However, we should note that our conclusion is not based on low risk-high risk considerations, but on closed economy-open economy considerations. In addition, our result hinges on the sign of the parameter γ again: we get the opposite result about welfare if $\gamma > 0$, for example.

2.4 The optimal size of the public sector

We have so far analyzed the equilibrium of the world economy assuming an exogenous size of the public sector. Now we obtain the size of the public sector that maximizes the welfare of the domestic representative agent or, for short, the optimal size of the public sector. We discuss whether maximizing welfare implies maximizing growth. Then we analyze the effect of changes in exogenous parameters on the optimal size of the public sector, on consumption-wealth ratio, on growth, and on welfare, provided that the size of the public sector is optimal. Finally, the results of an open economy are compared to those of a closed economy.

Formally, the expression in the right hand side of the Bellman equation (2.14) is partially differentiated with respect to g, where $G = gW$, to calculate the optimal size of the public sector

$$\frac{\eta}{g} C^{\gamma} (gW)^{\eta\gamma} - V'(W)W = 0,$$

which combining with the first order condition equation (2.15) implies that the optimal size of the public sector, $\widehat{g}$, must satisfy the following condition

$$\hat{g} = \eta \frac{C}{W}, \tag{2.41}$$

which is identical to Turnovsky (1996, p. 60; 1999, p. 888).[13] Equation (2.41) implies that the marginal utility of public consumption must be equal to the marginal utility of private consumption when both public and private consumption are optimally chosen.

Combining equation (2.41) with (2.22) we can calculate the optimal size of the public sector, the consumption-wealth ratio, and the growth rate when public consumption is optimally chosen in an open economy

$$\begin{aligned}
\hat{g}_o &= \frac{\eta}{[1-\gamma(1+\eta)](1+\eta)} \{\beta - \gamma(1+\eta)\rho \\
&\quad +0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma_{w,o}^2\} \\
\left(\frac{C}{W}\right)_o &= \frac{1}{[1-\gamma(1+\eta)](1+\eta)} \{\beta - \gamma(1+\eta)\rho \\
&\quad +0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma_{w,o}^2\} \\
\psi_o &= \frac{1}{1-\gamma(1+\eta)} \left\{\rho - \beta - 0.5\gamma(1+\eta)[1-\gamma(1+\eta)]\sigma_{w,o}^2\right\}.
\end{aligned} \tag{2.42}$$

[13] We should note that the optimal size of the public sector, $\hat{g}$, is not exactly identical to that shown in Turnovsky (1999). However, it is identical in the sense that in both cases the optimal ratio of public consumption to private consumption is given by $G/C = \eta$.

Do note that whenever we refer to the optimal size of the public sector in general we will use the term $\widehat{g}$ and whenever we refer only to the optimal size in an open economy we will use $\widehat{g}_o$.

In addition, we obtain the optimal size of the public sector, the consumption-wealth ratio, and the growth rate rate when public consumption is optimally chosen in a closed economy

$$\begin{aligned}
\hat{g}_c &= \frac{\eta}{[1-\gamma(1+\eta)]\,(1+\eta)}\{\beta-\gamma(1+\eta)\alpha \\
&\quad +0.5\gamma(1+\eta)\,[1-\gamma(1+\eta)]\,\sigma^2_{w,c}\} \\
\left(\frac{C}{W}\right)_c &= \frac{1}{[1-\gamma(1+\eta)](1+\eta)}\{\beta-\gamma(1+\eta)\alpha \\
&\quad +0.5\gamma(1+\eta)\,[1-\gamma(1+\eta)]\,\sigma^2_{w,c}\} \\
\psi_c &= \frac{1}{1-\gamma(1+\eta)}\left\{\alpha-\beta-0.5\gamma(1+\eta)\,[1-\gamma(1+\eta)]\,\sigma^2_{w,c}\right\}
\end{aligned} \tag{2.43}$$

Finally, in the case of a logarithmic utility function we find that the optimal size of the public sector is given by

$$\hat{g} = \frac{\eta\beta}{(1+\eta)}, \tag{2.44}$$

which is equal to the (deterministic) optimal size of the public sector obtained by Turnovsky (1996, p. 60) and very similar to Turnovsky (1999, p. 888). It is then easy to show that private consumption-wealth ratio plus the optimal size of the public sector is given by

$$\frac{C}{W} + \hat{g} = \beta, \tag{2.45}$$

where optimal consumption-wealth ratio is given by equation (2.35) above. Therefore, we obtain standard results in the literature again: (private plus public) consumption-wealth ratio is equal to the subjective discount rate.

2.4.1 Growth vs. welfare maximizing

Now we can compare the optimal size of the public sector with the size that maximizes the growth rate. Going back to equation (2.26) it is straightforward to calculate that the size of the public sector that maximizes the growth rate is zero. The intuition behind the result is immediate. Public spending is utility-enhancing but it does not affect the productivity of the economy. Therefore, since public spending does not enhance growth directly but it imposes a sacrifice, then the size of the public sector that maximizes growth should be zero. The optimal size of the public sector is clearly higher than the size that maximizes the growth rate. Both objectives are not equivalent.

2.4.2 Analysis of the optimal size

First, we focus on the influence of changes in exogenous variables that do not refer either to risk or the public sector. Differentiating equation (2.42) with respect to β

$$\frac{\partial \hat{g}_o}{\partial \beta} = \frac{\eta}{\left[1-\gamma(1+\eta)\right](1+\eta)} > 0,$$

we can observe that a higher subjective discount rate increases the optimal size of the public sector, because public consumption becomes more attractive. In addition, the effect of a higher gross rate of return, ρ, on the optimal size of the public sector is given by the expression

$$\frac{\partial \hat{g}_o}{\partial \rho} = -\frac{\eta\gamma}{\left[1-\gamma(1+\eta)\right]},$$

where a higher gross rate of return of the asset portfolio will raise the optimal size of the public sector for $\gamma < 0$. An increase in the gross rate of return originates a positive income effect on public consumption (allowing to dedicate more resources to public consumption) stronger than the negative substitution effect (public consumption becoming less attractive while investing more attractive).

Second, we analyze the impact of changes in exogenous variables that are related to risk, but not related to the behavior of the public sector. An increase in the parameter γ causes an ambiguous effect on the optimal size of the public sector. In addition, differentiating (2.42) with respect to $\sigma^2_{w,o}$

$$\frac{\partial \widehat{g}_o}{\partial \sigma^2_{w,o}} = 0.5\gamma\eta, \tag{2.46}$$

we show that a higher variance in the growth rate, $\sigma^2_{w,o}$, reduces the optimal size of the public sector if $\gamma < 0$. A higher variance in the growth rate is equivalent to a fall in the gross rate of return of the asset portfolio, ρ, as we showed above. That conclusion can be easily extended for the impact of a higher variance in domestic productivity shocks, σ^2_y, a higher covariance between domestic and foreign productivity shocks, σ_{yy^*}, or a higher variance in foreign productivity shocks, $\sigma^2_{y^*}$. For example, if shocks become less idiosyncratic in the EMU (that is, σ_{yy^*} increases) then the optimal size of the public sector should be lower for $\gamma < 0$. These results are in clear contrast to those found in Turnovsky (1999, pp. 888-889). He finds that, for a logarithmic utility function, a higher domestic risk increases unambiguously the optimal size of the public sector, whereas the impact of a higher foreign risk depends on whether the domestic economy holds positive stocks of foreign capital or not.

Third, we focus on the impact of changes in variables related to the behavior of the public sector. Then we easily show that whatever increases the variance of the growth rate, be a higher variance of domestic public spending, σ^2_z, be a lower covariance between domestic (foreign) productivity shocks and domestic public spending, σ_{yz} (σ_{y^*z}), should reduce the optimal size of the public sector if $\gamma < 0$, as we showed in (2.46).

Finally, focusing on the logarithmic case, we find that differentiating equation (2.44) with respect to the parameter η we obtain that

$$\frac{\partial \hat{g}}{\partial \eta} = \frac{\beta}{(1+\eta)^2} > 0, \tag{2.47}$$

which intuitively seems straightforward.

2.4.3 Consumption and growth

If we analyze the influence of changes in different exogenous parameters on consumption-wealth ratio and growth when the size of the public sector is optimal, then most of the qualitative results obtained when the size of the public sector was exogenously given do not change at all, even though the quantitative results do change. However, some results deserve attention.

Restricting ourselves to the case of a logarithmic utility function, it can be easily shown that an increase in the parameter η, in addition to raising the optimal size of the public sector unambiguously [see equation (2.47)], reduces in the same amount the private consumption-wealth ratio, given by equation (2.35). Going back to equation (2.45) above, a change in the parameter η modifies the distribution of total consumption spending between private and public spending. Increasing the optimal size of the public sector "crowds out" private consumption-wealth ratio one-to-one. Only variations in the subjective discount rate change private plus public consumption-wealth ratio. Changes in any other variable do not modify either total consumption spending or the distribution between both types of consumption. Therefore, a change in the parameter η does not change the growth rate of wealth, in contrast to the conclusions we got in Section 2.3.2 above.

2.4.4 Open economy versus closed economy

Now we can compare the optimal size of the public sector in an open economy to that in a closed economy, as well as the consumption-wealth ratio, the growth rate, and welfare in the same way we did when the size of the public sector was exogenously given in Section 2.3.4.

First, if we substract equation (2.43) from equation (2.42) we obtain using equation (2.37), after some algebra, that

$$\hat{g}_o - \hat{g}_c = -0.5\eta\gamma\Delta n_d^{*^2}.$$

We focus on the case $n_d = \widetilde{n}_d$, where $\widetilde{n}_d$ is the variance-minimizing share of the domestic portfolio, given by equation (2.39). However, the results do not depend on that assumption. Then we obtain from equation (2.37) that the variance in the growth rate in an open economy is lower than that in a closed economy, $\sigma_{w,o}^2 < \sigma_{w,c}^2$. As we saw above, a reduction of the variance in the growth rate is equivalent to an increase in the gross rate of return of the asset portfolio. That, in turn, originates a stronger positive income effect than the negative substitution effect: the optimal size of the public sector in an open economy is higher than in a closed economy, which is what the empirical evidence suggests (Rodrik, 1998). In addition, the higher the value of the optimal share of the domestic portfolio materialized in foreign capital, n_d^*, the higher the difference between the optimal size of the public sector in an open economy with that in a closed economy. The result we have obtained is equal to that shown in Turnovsky (1999), but differs significantly in the conditions that are necessary to reach that conclusion: we get that result for $\gamma < 0$, which is what the empirical evidence suggests, and he shows that the optimal size in an open economy is higher than in a closed economy if the utility function is logarithmic, provided that the domestic economy holds positive stocks of foreign capital in a small open economy.

Second, similarly we can obtain the difference between the consumption-wealth in an open economy compared to that in a closed economy

$$\left(\frac{C}{W}\right)_o - \left(\frac{C}{W}\right)_c = -0.5\gamma\Delta n_d^{*^2}.$$

We can show again if $\gamma < 0$ then the consumption-wealth ratio in an open economy is higher than in a closed economy. Therefore, provided that the size of the public sector is optimal, we get the same qualitative results as those obtained when the size of the public sector was exogenously given. However, the quantitative results do change slightly.

Third, comparing the growth rate in an open economy to that in a closed economy, the results will be qualitatively identical to those obtained in the case where the size of the public sector was exogenously given. Thus, we shall not pursue the analysis further.

Finally, we can easily show that welfare is higher in a risky open economy than in a risky closed economy if $\gamma < 0$, as we showed above in Section 2.3.4.

2.5 Conclusions

The impact of risk and utility-enhancing public spending on the economy is a topic that has been analyzed extensively. However, the models have been focused almost exclusively on closed or small open economies. In this paper we have analyzed a two-country stochastic AK growth model, based on Turnovsky (1997, Ch. 11), where the consumption good provided by the public sector is utility-enhancing [Barro (1990)]. The results obtained can be divided into four groups.

First, having characterized the world equilibrium, we have analyzed the impact of changes in different exogenous variables on the consumption-wealth ratio, the growth rate of wealth, and welfare. Most of the results are standard in the literature. However, we have shown that a higher weight of public consumption in the utility function raises the growth rate, due to a reduction in the consumption-wealth ratio, given the exogenous size of the public sector. Therefore, different preferences towards utility-enhancing government expenditure produce different growth rates, other things being equal. In addition, even though increasing the size of the public sector is always growth-reducing, it is welfare-augmenting when the size of the public sector is below its optimal size.

Second, we have compared the results in an open economy with those of a closed economy. Thus we have shown that consumption-wealth ratio in an open economy should be higher than that in a closed economy. In the simplest case where the portfolio share is equal to that which minimizes the variance of the growth rate, an open economy achieves a lower variance of the growth rate thus encouraging consumption. Then, we have discussed whether the growth rate of assets in an open economy should be higher than in a closed economy. Even though the model does not offer clear-cut results, we have shown that an open economy will unambiguously grow slower than a closed economy if the marginal physical product of domestic capital is higher than or equal to that of foreign capital, for instance. In addition, since welfare depends basically on the consumption-wealth ratio, welfare is higher in a risky open economy than in a risky closed economy, thus extending the results in Obstfeld (1994) and Turnovsky (1997, Ch. 11).

Third, we have derived the welfare-maximizing size of the public sector and compared it to the size that maximizes growth. Then we have analyzed the impact of changes in different exogenous variables on the optimal size of the public sector. Thus we have shown that whatever increases the variance of the growth rate (a higher covariance between domestic and foreign productivity shocks, for example) reduces the optimal size of the public sector, in contrast to the results found in Turnovsky (1999). In addition, a higher value of the parameter η increases the optimal size of the public sector just in the same amount private consumption-wealth ratio falls, so that public plus private consumption-wealth ratio and the growth rate of wealth do not change. That changes substantially our conclusions with respect to those when the size of the public sector was exogenously given. Next, we have established that the optimal size of the public sector in an open economy is higher than that in a closed economy under more general conditions than those established in Turnovsky (1999). The lower variance of the growth rate obtained in an open economy tends to raise public consumption.

Finally, we should point out possible avenues for future research. The assumption of continuous budget equilibrium could be relaxed, thus introducing public bonds in the model. However, that would increase enormously the complexity of the model. Introducing money is also an interesting element that could be integrated into a two-country world economy. Additionally, public spending could be productive also, and not only utility enhancing.

Chapter 3

Risk, productive government expenditure, and the world economy

3.1 Introduction

The impact of government expenditure policy on long run growth is an important policy issue. The emergence of endogenous growth models in the 80's have provided a useful approach to analyze how government expenditure policy can influence on the long run trajectory of the economy. Barro (1990) pioneered the analysis based on a closed economy deterministic AK growth model where public spending is productive[1]. Others, Turnovsky (1998, 1999) for instance, have followed suit incorporating small open economy features, risk and other issues (such as congestion, for example) into endogenous growth models where public spending is productive. Thus substantial conclusions have been derived concerning the impact of risk and the expenditure policy of the public sector on the economy, and the optimal size of the public sector, provided that public spending enhances productivity and volatility. However, there is a recurrent shortage of analysis based on two-country stochastic models, specially when the integration of financial markets is becoming more complete.

This paper analyzes the influence of risk and the expenditure policy of the public sector by incorporating productive public spending [see Barro (1990)]

[1] Barro (1990) also analyzed the role of utility-enhancing government expenditure.

into a two-country stochastic AK growth model developed by Turnovsky (1997, Ch. 11). Thus, we derive the size of the public sector that maximizes welfare endogenously, instead of assuming an exogenous size, as in Turnovsky (1997, Ch. 11). Previous analysis introduced risk into endogenous growth models, but public spending was neither utility-enhancing nor productive [see Eaton (1981), for example]. Turnovsky extended Barro's (1990) closed economy model by introducing productivity- and volatility-enhancing public spending into a stochastic endogenous growth small open economy. In addition, Turnovsky (1998) analyzed the impact of productive public spending (subject to congestion) in a risky closed economy. Therefore, our model has been constructed combining the main characteristics of the core literature[2]:

- It is an AK growth model, as the rest of the models.

- It is a two-country model, following the framework set out by Turnovsky (1997, Ch. 11), whereas the rest are one-country models (either a closed economy or a small open economy).

- Public spending is productive, pioneered by Aschauer (1989) and incorporated originally into endogenous growth models by Barro (1990). Thus, the model is able to determine the size of the public sector that maximizes the welfare of the representative agent, as most of the models in the core literature do. Turnovsky (1997) is the only model that cannot analyze the magnitude of such a size, since public spending is neither utility enhancing nor productive, so "it can be interpreted as being a real drain on the economy or, alternatively, as some public good that does not affect the marginal utility of private consumption or the productivity of private capital" (Turnovsky, 1997, p. 338). Turnovsky (1998, 1999) extends Barro (1990) from a closed economy to a model with congestion and to a small open economy setting, respectively.

- The model is stochastic. Barro (1990) is the only model of the core literature that is not stochastic. Turnovsky (1998, 1999) extend the deterministic model in Barro (1990) to a stochastic setting.

[2]The term "core literature" refers to the model developed by Turnovsky (1997, Ch. 11) and the papers that have analyzed the impact of risk and/or the expenditure policy of the public sector on long term growth based on AK growth models, provided that public spending is productive and no congestion arises.

Table 3.1: An overview of the model

Existing models	AK growth	Two countries	Size of the public sector	Stochastic shocks
Barro (1990)	X		X	
Turnovsky (1997)	X	X		X
Turnovsky (1998)	X		X	X
Turnovsky (1999)	X		X	X
This model	X	X	X	X

Table 3.1 encompasses the relationship between the model of this paper and the core literature.

This model can be specially useful at the present moment in the European Economic and Monetary Union (EMU). First, countries of the euro area have adopted the Stability and Growth Pact (SGP) from 1st January 1999 onwards. The objective of the SGP is that countries within the euro area must attain budget balance, in the medium or in the long run, so the assumption of continuous budget balance that we make in this chapter seems reasonable. Second, the emphasis of this paper is the long run and, therefore, it does not focus on the influence of business cycles, important as they may be. Finally, there is a permanent debate about whether the size of the public sector should be bigger or smaller and, more specifically, whether more open economies should have bigger governments or not. This model sheds some light on the issue, since it compares the size of the public sector that maximizes welfare in an open economy with that in a closed economy.

This paper is organized as follows. We first obtain the world macroeconomic equilibrium, given that the size of the public sector is exogenously given. Then, we study the impact of changes in exogenous variables on key economic variables such as consumption-wealth ratio, the growth rate of wealth and welfare. The results derived from an open economy are compared to those of a closed economy. Next, the welfare-maximizing size of the public sector is derived. The differences arising from maximizing growth and welfare are discussed. Additionally, we analyze whether more open economies are associated with a higher size optimal of the public sector, even when public spending is productive-only. Finally, we conclude by indicating possible avenues for future research.

3.2 The world economy

3.2.1 Basic structure

The world is a real economy composed of two countries, each of them producing only one homogeneous good. In each country exists a representative agent with an infinite time horizon. The homogeneous good produced by both countries can be either consumed or invested in capital without having to incur in any kind of adjustment costs. There are two assets: domestic capital and foreign capital. Unstarred variables refer to domestic economy, whereas starred variables refer the foreign economy. Both domestic capital, K, and foreign capital, K^*, can be owned by the domestic representative agent or the foreign representative agent. The subscript d denotes the holdings of assets of the domestic representative agent and the subscript f denotes the holdings of assets of the foreign representative agent. So it must be satisfied that

$$\begin{aligned} K &= K_d + K_f \\ K^* &= K_d^* + K_f^*. \end{aligned}$$

Therefore, the wealth of the domestic representative agent, W, and the wealth of the foreign representative agent, W^*, will be

$$W = K_d + K_d^* \tag{3.1}$$

$$W^* = K_f + K_f^*. \tag{3.2}$$

The public sector purchases part of the private flow of production and utilizes it to supply a productive pure public good to the private representative agent. Public spending, dG, increases with wealth, so we can achieve a balanced growth path[3]. We specify public spending as follows

$$dG = gWdt + Wdz, \tag{3.3}$$

[3] Other rules can also achieve a balanced growth path. See Turnovsky (1996) for more details.

where $g = G/W$ denotes the size of the public sector, and dz is the increment of a stochastic process z. Those increments are temporally independent and are normally distributed. They satisfy that $E(dz) = 0$ and $E(dz^2) = \sigma_z^2 dt$.

Domestic production can be obtained using only domestic capital, K, through a somewhat modified AK function and it is expressed through a first order stochastic differential equation

$$dY = \alpha K dt + \alpha K dy, \tag{3.4}$$

where

$$\alpha = \overline{\alpha} + \delta g - 0.5\theta g^2. \tag{3.5}$$

The term $\overline{\alpha} > 0$ is the (constant) physical marginal product of private capital when the size of the domestic public sector is zero and dy represents a proportional domestic productivity shock. More precisely, dy is the increment of a stochastic process y. Those increments are temporally independent and are normally distributed. They satisfy that $E(dy) = 0$ and $E(dy^2) = \sigma_y^2 dt$.[4] We omit, for convenience, formal references to time, although those variables depend on time. We must note that dY indicates the flow of production, instead of Y, as is ordinarily done in stochastic calculus.

The production function incorporates the influence of the public sector on the physical marginal product of private capital and on the magnitude of the stochastic domestic productivity shock by means of a quadratic term in g. The modified marginal physical product of private capital, α, is based on Gallaway and Vedder (1995) and Vedder and Gallaway (1998, p. 4). Vedder and Gallaway (1998) refer to what US Congress Representative Richard Armey (1995) termed the Armey Curve, which is a figure *à la Laffer* relating the size of the public sector to the growth rate of the economy (Vedder and Gallaway, 1998, p. 1). However, the curve relating both variables can, in fact, be found before in Barro (1990, pp. S110 and S118), or in Barro and Sala-i-Martin (1995, p. 155) and thus the Armey Curve should be more conveniently renamed as the Barro-Sala-i-Martin-Armey (BSiMA) Curve. Here we have converted that relationship into one between the size of the public sector and the marginal physical product of private capital.

[4] That is, the production flow follows a Brownian motion with drift αK and with variance $\alpha^2 K^2 \sigma_y^2$.

Both parameters δ and θ in equation (3.5) are positive, so that the function is concave in the size of the public sector, g, and we restrict ourselves to the case $g < \delta/\theta < 1$. Then, we assume that the marginal impact of the public sector on the marginal physical product of private capital is positive, at a diminishing rate[5]. In addition, an increase in the size of the public sector amplifies the magnitude of the impact of domestic productivity shocks, at a diminishing rate. We could easily introduce into the model the alternative assumption that an increase in the size of the public sector reduces the impact of domestic productivity shocks, just changing the signs for the parameters δ and θ for the stochastic component in equation (3.5) above. In case $\delta = \theta = 0$, we return to a standard AK production function.

This production function captures essentially, albeit in an different alternative way, the basic features of models with a productive public sector, such as Barro (1990), or Turnovsky (1998, 1999) in stochastic settings, and it makes possible to extend the analysis and to compare our results to theirs. We have chosen this alternative way of modeling because it is easier to adapt to a two-country world economy and, additionally, it can be easily extended to encompass volatility-reducing features. We should note that here we introduce the flow of production goods provided by the public sector and not the stock of accumulated public capital stock.[6] Thus, even though we should postulate it as a stock (being the spending in public physical structures), that would lead to a transitional dynamics equilibrium (Turnovsky, 1998, p. 6). The literature has usually opted to postulate it as a flow to be analytically tractable.[7] Additionally, we should note that our formulation implies that only the deterministic component of public spending in production goods is productive.

The same structure applies to the foreign economy. Foreign public spending is given by

$$dG^* = g^* W^* dt + W^* dz^*,$$

[5]It can be easily obtained that the marginal physical product of private capital, α, becomes a maximum when $g = \delta/\theta$. Here we are assuming that the marginal impact of public spending on α becomes negative for some value of the size of the public sector $g < 1$ and, therefore, that it is negative for $g = 1$ as well.

[6]As Turnovsky puts it (1998, p. 5), "In introducing productive government expenditure one must choose between formulating it as a flow or as a stock, a choice that involves a tradeoff between tractatibility and realism".

[7]See, for example, Barro (1990) and Turnovsky (1998, 1999).

where $g^* = G^*/W^*$ denotes the size of the foreign public sector, and dz^* is the increment of a stochastic process z^*. Those increments are temporally independent and are normally distributed. They satisfy that $E(dz^*) = 0$ and $E(dz^{*^2}) = \sigma_{z^*}^2 dt$.

The foreign economy is structured symmetrically to the domestic economy. Thus, foreign production is obtained using only foreign capital, K^*, through a modified AK function

$$dY^* = \alpha^* K^* dt + \alpha^* K^* dy^*,$$

such that

$$\alpha^* = \overline{\alpha}^* + \delta^* g^* - 0.5\theta^* g^{*^2},$$

where $\overline{\alpha}^* > 0$ is the marginal physical product of capital when the size of the foreign public sector is zero and dy^* represents a proportional foreign productivity shock. The term dy^* is the increment of a stochastic process y^*. Those shocks are temporally independent and are distributed normally, satisfying that $E(dy^*) = 0$ and that $E(dy^{*^2}) = \sigma_{y^*}^2 dt$.

3.2.2 Domestic economy

The maximization problem

The preferences of the domestic representative agent are represented by an isoelastic intertemporal utility function where she obtains utility from consumption, C

$$E_0 \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\beta t} dt; -\infty < \gamma < 1. \tag{3.6}$$

The welfare of the domestic representative agent in period 0 is the expected value of the discounted sum of instantaneous utilities, conditioned on the set of disposable information in period 0. Parameter β is a positive subjective discount rate (or rate of time preference). The utility function becomes logarithmic when $\gamma = 0$. The empirical evidence suggests a high degree of risk aversion (Campbell, 1996).[8] Restrictions on the utility function are necessary to ensure concavity with respect to consumption.

The domestic representative agent consumes at a deterministic rate $C(t)dt$ in the instant dt and must pay the corresponding taxes and thus the dynamic budget restriction can be expressed as

$$dW = [\alpha K_d + \alpha^* K_d^*]\, dt + [\alpha K_d dy + \alpha^* K_d^* dy^*] - Cdt - dT, \tag{3.7}$$

where dT denotes the taxes the domestic representative agent must pay to the public sector. We assume that the collection of taxes exactly offsets public spending

$$dT = dG, \tag{3.8}$$

that is, the public sector balances budget continuously.

Combining equations (3.3) and (3.8), and plugging them into (3.7), the restriction for the resources of the domestic economy is given by

$$dW = [\alpha K_d + \alpha^* K_d^* - C - gW]\, dt + [\alpha K_d dy + \alpha^* K_d^* dy^* - W dz]\,. \tag{3.9}$$

Going back to equation (3.1), if we define the following variables for the domestic representative agent

$$\begin{aligned} n_d &\equiv \frac{K_d}{W} = \text{ share of the domestic portfolio materialized} \\ &\quad \text{in domestic capital} \\ n_d^* &\equiv \frac{K_d^*}{W} = \text{ share of the domestic portfolio materialized} \\ &\quad \text{in foreign capital,} \end{aligned}$$

equation (3.1) can be expressed in a more convenient way as

$$1 = n_d + n_d^*$$

Substituting those variables into the budget constraint (3.9) we obtain the following dynamic restriction for the resources of the domestic economy

$$\frac{dW}{W} = \left[\alpha n_d + \alpha^* n_d^* - \frac{C}{W} - g\right] dt + [\alpha n_d dy + \alpha^* n_d^* dy^* - dz]\,.$$

[8] The Arrow-Pratt coefficient of risk aversion is given by $1 - \gamma$.

This equation can be more conveniently expressed as

$$\frac{dW}{W} = \psi dt + dw, \tag{3.10}$$

where the deterministic and stochastic parts of the accumulation rate of assets, dW/W, can be expressed in the following way

$$\begin{aligned} \psi &\equiv n_d [\alpha - \alpha^*] + \alpha^* - g - \frac{C}{W} \equiv \rho - g - \frac{C}{W} & (3.11) \\ dw &\equiv n_d [\alpha dy - \alpha^* dy^*] + \alpha^* dy^* - dz, & (3.12) \end{aligned}$$

where $\rho \equiv \alpha n_d + \alpha^* n_d^* \equiv n_d [\alpha - \alpha^*] + \alpha^*$ denotes the gross rate of return of the asset portfolio.

Equilibrium

The objective of the domestic representative agent is to choose the path of consumption and portfolio shares that maximizes the expected value of the intertemporal utility function (3.6), subject to $W(0) = W_0$, (3.10), (3.11) and (3.12). This optimization is a stochastic optimum control problem.[9] Initially, we assume that the public sector sets an arbitrarily exogenous size of the public sector, g. We analyze the case in which such a size will be chosen optimally in section 3.4.

It is important to bear in mind that the domestic agent takes as given the rates of return of different assets, as well as the corresponding variances and covariances. However, these parameters will endogenously be determined in the macroeconomic equilibrium we shall obtain. We look for values of the endogenous variables that are not stochastic in equilibrium, and then, we show that the results validate the initial assumption that equilibrium values are not stochastic.

We introduce a value function, $V(W)$, which is defined as

$$V(W) = Max_{\{C,n_d\}} \;\; E_0 \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\beta t} dt, \tag{3.13}$$

[9] To solve problems of stochastic optimum control see, for example, Kamien and Schwartz (1991, section 22), Malliaris and Brock (1982, ch. 2), Obstfeld (1992), or Turnovsky (1997, ch. 9; 2000, ch. 15).

subject to restrictions (3.10), (3.11) and (3.12), and given initial wealth. The value function in period 0 is the expected value of the discounted sum of instantaneous utilities, evaluated along the optimal path, starting in period 0 in the state $W(0) = W_0$.

Starting from equation (3.13) the value function must satisfy the following equation, known as the Hamilton-Jacobi-Bellman equation of stochastic control theory or, for short, the Bellman equation

$$\beta V(W) = Max_{\{C,n_d\}} \quad \left[\frac{1}{\gamma}C^{\gamma} + V'(W)W\psi + 0.5V''(W)W^2\sigma_w^2\right], \qquad (3.14)$$

where ψ can be found in (3.11) and σ_w^2 denotes the variance of the stochastic element of accumulation rate of assets, given by equation (3.12).

Equation (3.14) is differentiated partially with respect to C and n_d in order to obtain the first order conditions of this optimization

$$C^{\gamma-1} - V'(W) = 0 \qquad (3.15)$$

$$V'(W)W(\alpha - \alpha^*) + V''(W)W^2 cov\left[dw, \alpha dy - \alpha^* dy^*\right] \;=\; 0. \qquad (3.16)$$

The solution to this problem is obtained through trial and error. We seek to find a value function $V(W)$ that satisfies, on the one hand, the first order optimality conditions and, on the other, the Bellman equation. In the case of isoelastic utility functions, the value function has the same form as the utility function [Merton (1969), result generalized in Merton (1971)]. Thus, we guess that the value function is of the form

$$V(W) = AW^{\gamma}, \qquad (3.17)$$

where the coefficient A will be determined below. This guess implies that

$$\begin{aligned} V'(W) &= A\gamma W^{\gamma-1} \\ V''(W) &= A\gamma(\gamma-1)W^{\gamma-2}. \end{aligned}$$

Plugging these expressions into the first order optimality conditions (3.15) and (3.16) we obtain

$$C^{\gamma-1} = A\gamma W^{\gamma-1} \tag{3.18}$$

$$(\alpha - \alpha^*)\, dt = (1-\gamma)\, cov\left[dw, \alpha dy - \alpha^* dy^*\right] \quad . \tag{3.19}$$

These are typical equations in stochastic models in continuous time. Equation (3.18) indicates that at the optimum, the marginal utility derived from consumption must be equal to the marginal change in the value function or the marginal utility of wealth. Equation (3.19) shows that the optimal choice of portfolio shares of the domestic representative agent must be such that the risk-adjusted rates of return of both assets are equalized.

Combining (3.18) and (3.19), and inserting them into the equation (3.14), we can calculate, after some algebra, the equilibrium portfolio shares and the consumption-wealth ratio in the domestic open economy

$$n_d = \frac{\alpha - \alpha^*}{(1-\gamma)\,\Delta} + \frac{{\alpha^*}^2 \sigma_{y^*}^2 - \alpha\alpha^*\sigma_{yy^*} + \alpha\sigma_{yz} - \alpha^*\sigma_{y^*z}}{\Delta} \tag{3.20}$$

$$n_d^* = 1 - n_d$$

$$\left(\frac{C}{W}\right)_o = \frac{1}{1-\gamma}\left\{\beta - \gamma(\rho - g) + 0.5\gamma(1-\gamma)\,\sigma_{w,o}^2\right\}, \tag{3.21}$$

where

$$\Delta = \alpha^2\sigma_y^2 - 2\alpha\alpha^*\sigma_{yy^*} + {\alpha^*}^2\sigma_{y^*}^2 \tag{3.22}$$

$$\begin{aligned}\sigma_{w,o}^2 = {} & n_d^2\alpha^2\sigma_y^2 + 2n_d n_d^*\alpha\alpha^*\sigma_{yy^*} + {n_d^*}^2{\alpha^*}^2\sigma_{y^*}^2 + \sigma_z^2 \\ & -2n_d\alpha\sigma_{yz} - 2n_d^*\alpha^*\sigma_{y^*z}.\end{aligned} \tag{3.23}$$

Please note that neither Δ nor the variance of the growth rate of assets, $\sigma_{w,o}^2$, can be negative and the subscript o refer to values in an open economy.

The equilibrium is characterized by a balanced real growth. The equilibrium rate of wealth accumulation of the domestic economy follows the stochastic process

$$\frac{dW}{W} = \psi_o dt + dw_o,$$

where the deterministic and stochastic components are, respectively

$$\psi_o = \frac{1}{1-\gamma}\left\{\rho - g - \beta - 0.5\gamma\left(1-\gamma\right)\sigma_{w,o}^2\right\} \tag{3.24}$$

$$dw_o = n_d \alpha dy + n_d^* \alpha^* dy^* - dz. \tag{3.25}$$

Now we can obtain the equilibrium solution in a closed economy by setting $n_d = 1$ and $n_d^* = 0$ in equations (3.21), (3.23), (3.24), and (3.25). We shall use the shares of the domestic portfolio materialized in domestic and foreign capital, n_d and n_d^* respectively, to approximate the degree of openness of the domestic economy. The equilibrium of the domestic economy if it were closed is given by

$$\left(\frac{C}{W}\right)_c = \frac{1}{1-\gamma}\left\{\beta - \gamma\left(\alpha - g\right) + 0.5\gamma\left(1-\gamma\right)\sigma_{w,c}^2\right\} \tag{3.26}$$

$$\sigma_{w,c}^2 = \alpha^2\sigma_y^2 + \sigma_z^2 - 2\alpha\sigma_{yz} \tag{3.27}$$

$$\psi_c = \frac{1}{1-\gamma}\left\{\alpha - g - \beta - 0.5\gamma\left(1-\gamma\right)\sigma_{w,c}^2\right\} \tag{3.28}$$

$$dw_c = \alpha dy - dz,$$

where the variables with the subscript c refer to values in a closed economy. We should note that in case of no risk, that is, $\sigma_{w,c}^2 = 0$, by differentiating equation (3.28) with respect to the size of the public sector, g, we can see that the growth rate of assets first increases with the size of the public sector but then diminishes with the size of the public sector, thus implying the BSiMA Curve referred above.

In order to guarantee that consumption is positive in the domestic open economy we impose the feasibility condition that the marginal propensity to consume out of wealth must be positive since wealth does not become negative

$$\frac{1}{1-\gamma}\left\{\beta - \gamma\left(\rho - g\right) + 0.5\gamma\left(1-\gamma\right)\sigma_{w,o}^2\right\} > 0.$$

For the first order optimality conditions to characterize a maximum, the corresponding second order condition must be satisfied, that is, the Hessian matrix associated to the maximization problem and evaluated at the optimal values of the choice variables

$$\begin{bmatrix} (\gamma-1)\left(V'(W)\right)^{\frac{\gamma-2}{\gamma-1}} & 0 \\ 0 & V''(W)W^2\Delta \end{bmatrix}$$

must be negative definite,[10] which implies that

$$\begin{aligned} (\gamma-1)\left(V'(W)\right)^{\frac{\gamma-2}{\gamma-1}} &< 0 \\ V''(W)W^2\Delta &< 0, \end{aligned}$$

where $\Delta > 0$ (in a risky economy) was already defined in equation (3.22). To evaluate those conditions we first obtain the value of the coefficient A in equation (3.18)

$$A = \frac{1}{\gamma}\left(\frac{C}{W}\right)^{\gamma-1}, \tag{3.29}$$

where C/W is the optimal value pointed out by equation (3.21). Then, we substitute (3.29) into the value function (3.17). Then the value function is given, after some algebra, by

$$V(W) = \frac{1}{\gamma}\left(\frac{C}{W}\right)^{\gamma-1} W^{\gamma}, \tag{3.30}$$

where we can observe that, given the restrictions on the utility function, $V'(W) > 0$ and $V''(W) < 0$ provided that $C/W > 0$.

In addition, we impose that the macroeconomic equilibrium must satisfy the transversality condition so as to guarantee the convergence of the value function

$$\lim_{t\to\infty} E\left[V\left(W\right)e^{-\beta t}\right] = 0. \tag{3.31}$$

Now, let us show that should the feasibility condition be satisfied, that would be equivalent to satisfying the transversality condition.[11] To evaluate (3.31),

[10] See Chiang (1984, pp. 320-323), for example.

[11] See Merton (1969). Turnovsky (2000) provides, for example, proof of the transversality condition as well.

we start expressing the dynamics of the accumulation of wealth

$$dW = \psi W dt + W dw. \tag{3.32}$$

The solution to equation (3.32), starting from the initial wealth $W(0)$, is[12]

$$W(t) = W(0)e^{(\psi - 0.5\sigma_w^2)t + w(t) - w(0)}.$$

Since the increments of w are temporally independent and are normally distributed then[13]

$$\begin{aligned} E[AW^{\gamma}e^{-\beta t}] &= E[AW(0)^{\gamma}e^{\gamma(\psi - 0.5\sigma_w^2)t + \gamma[w(t)-w(0)] - \beta t}] \\ &= AW(0)^{\gamma}e^{\left[\gamma(\psi - 0.5\sigma_w^2) + 0.5\gamma^2\sigma_w^2 - \beta\right]t}. \end{aligned}$$

The transversality condition (3.31) will be satisfied if and only if

$$\gamma\left\{\psi - 0.5\gamma\left(1-\gamma\right)\sigma_w^2\right\} - \beta < 0.$$

Now substituting equations (3.24) and (3.21), it can be shown that this condition is equivalent to

$$\frac{C}{W} > 0,$$

and thus feasibility guarantees convergence as well.

Finally, we should note that since the public sector equilibrates its budget continuously, the intertemporal budget constraint of the public sector is satisfied trivially.

3.2.3 Foreign economy

The maximization problem

The structure of the foreign economy and the problem facing the foreign representative agent can be formulated in an analogous way to the domestic

[12] See Malliaris and Brock (1982, pp. 135-136), for example.
[13] See Malliaris and Brock (1982, pp. 137-138), for example.

economy. Her preferences are represented by the following intertemporal utility function

$$E_0 \int_0^\infty \frac{1}{\gamma^*} C^{\gamma^*} e^{-\beta^* t} dt; -\infty < \gamma^* < 1.$$

The dynamics of foreign wealth are given by

$$\frac{dW^*}{W^*} = \psi^* dt + dw^*, \tag{3.33}$$

where

$$\psi^* \equiv n_f \alpha + n_f^* \alpha^* - g^* - \frac{C^*}{W^*} \equiv \rho^* - g^* - \frac{C^*}{W^*} \tag{3.34}$$

$$dw^* \equiv n_f \alpha dy + n_f^* \alpha^* dy^* - dz^*. \tag{3.35}$$

Equilibrium

The equilibrium portfolio shares and the consumption-wealth ratio in the foreign economy are given by

$$n_f = \frac{\alpha - \alpha^*}{(1-\gamma^*)\Delta} + \frac{{\alpha^*}^2 \sigma_{y^*}^2 - \alpha\alpha^* \sigma_{yy^*} + \alpha\sigma_{yz} - \alpha^* \sigma_{y^*z}}{\Delta}$$

$$n_f^* = 1 - n_f$$

$$\left(\frac{C^*}{W^*}\right)_o = \frac{1}{1-\gamma^*}\left\{\beta^* - \gamma^*(\rho^* - g^*) + 0.5\gamma^*(1-\gamma^*)\sigma_{w^*,o}^2\right\},$$

where

$$\sigma_{w^*,o}^2 = n_f^2 \alpha^2 \sigma_y^2 + 2 n_f n_f^* \alpha\alpha^* \sigma_{yy^*} + {n_f^*}^2 {\alpha^*}^2 \sigma_{y^*}^2 + \sigma_z^2 - 2 n_f \alpha \sigma_{yz} - 2 n_f^* \alpha^* \sigma_{y^*z},$$

and Δ was already defined in equation (3.22) above.

The equilibrium rate of wealth accumulation in the foreign economy follows the stochastic process

$$\frac{dW^*}{W^*} = \psi_o^* dt + dw^*,$$

where its deterministic and stochastic components are, respectively

$$\begin{aligned}\psi_o^* &= \frac{1}{(1-\gamma^*)}\left\{\rho^* - g^* - \beta^* - 0.5\gamma^*(1-\gamma^*)\sigma^2_{w^*,o}\right\} \\ dw_o^* &= n_f\alpha dy + n_f^*\alpha^* dy^* - dz^*.\end{aligned}$$

3.3 Equilibrium analysis

Now the impact of changes in exogenous variables on the consumption-wealth ratio, the growth rate of wealth of the domestic economy, and welfare, is briefly reviewed, since most of the results are standard.[14] Next, the results from an open economy are compared to those of a closed economy, provided that the size of the public sector is exogenously given.

3.3.1 Consumption

The optimal consumption-wealth ratio obtained in equation (3.21) is standard in the literature: domestic consumption is a linear function of domestic wealth.[15] To start with, we review the impact of changes in exogenous variables that are not directly related to risk or public spending on consumption. Thus, a higher subjective discount rate, β, increases consumption-wealth ratio, because the domestic representative agent finds more attractive to dedicate a higher proportion of wealth to consumption, thus reducing investment. In addition, the impact of a higher gross rate of return of the asset portfolio, ρ, on consumption-wealth ratio depends on the sign of the parameter γ. That is the overall result of two opposite effects, substitution and income effects. A higher gross rate of return of the asset portfolio has always a negative substitution effect since consumption becomes less attractive whereas investment is more attractive. The income effect on the consumption-wealth ratio originated by a higher gross rate of return of the asset portfolio is equal to unity: it makes possible to raise both actual and future consumption. For example, if $\gamma < 0$ income effect dominates substitution effect. Thus, increasing the gross rate of return of the asset portfolio, ρ, raises consumption-wealth ratio.

[14] See Turnovsky (1997, Ch. 11), for example.

[15] Pioneered by Merton (1969[1992]) within a context of uncertainty using in continuous time. We refer to Turnovsky (1997, 2000) for details more related to our model.

From here onwards whenever the result only depends on the sign of the parameter γ, we focus on the case where $\gamma < 0$, for being the most empirically relevant situation [Campbell (1996)].

Second, the impact of variables related to risk, but not affected by the behavior of the public sector, is reviewed. Thus the effect of a higher coefficient of risk aversion, γ, on consumption is ambiguous. In addition, a higher variance of the growth rate, $\sigma^2_{w,o}$, reduces consumption-wealth ratio if $\gamma < 0$. Substitution and income effects arise again: totally differentiating equation (3.21), it can be easily shown that an increase in the variance of the growth rate is equivalent to a fall in the gross rate of return of the asset portfolio, ρ, of $0.5\,[1-\gamma(1+\eta)]$. Similar conclusions apply to the impact of a higher variance of domestic productivity shocks, σ^2_y, a higher variance of foreign productivity shocks, $\sigma^2_{y^*}$, or a higher covariance between domestic and foreign productivity shocks, σ_{yy^*}, on consumption-wealth ratio.

Third, the role of the public sector is reviewed. A higher size of the public sector, g, originates a positive productive effect plus a negative volatility effect: the net effect depends on which of the effects dominate. For example, if the net effect is positive, which is equivalent to a rise in the gross rate of return of the asset portfolio, ρ, consumption-wealth ratio increases for $\gamma < 0$. Next, an increase in the variance of public spending shocks, σ^2_z, diminishes consumption-wealth ratio when $\gamma < 0$. An increase in the variance of public spending shocks is equivalent to a fall in the gross rate of return of the asset portfolio of $0.5\,[1-\gamma(1+\eta)]$, since the variance of the growth rate increases. In contrast, if either the covariance between domestic productivity shocks and domestic public spending shocks, σ_{yz}, or the covariance between foreign productivity shocks and domestic public spending shocks, σ_{y^*z}, increase, consumption-wealth ratio increases for $\gamma < 0$. That is due to a reduction in the variance of the growth rate of the domestic economy.

For the case that the utility function is logarithmic, that is, $\gamma = 0$, we find the familiar result $C/W = \beta$. Only changes in the subjective discount rate alter consumption-wealth ratio.

3.3.2 Growth

The equilibrium mean growth rate of assets, shown in (3.24), is standard in the literature[16]. First, we review the impact of variables that do not refer either to risk or public spending on the growth rate of assets. Thus, a higher subjective discount rate, β, reduces unambiguously the growth rate since dedicating resources to consumption becomes more attractive whereas investment is discouraged. In addition, a higher gross rate of return of the asset portfolio, ρ, increases the growth rate, even though consumption-wealth ratio may rise.

Second, we study the impact of variables related to risk, but not affected by the behavior of the public sector. Thus, a change in the parameter γ generates an ambiguous effect on the growth rate. Next, an increase in the variance of domestic productivity shocks, σ_y^2, shifting investment towards foreign capital, tends, on the one hand, to increase the growth rate if $\alpha^* > \alpha$. On the other hand, the growth-enhancing effect is reinforced when $\gamma < 0$ since consumption-wealth ratio falls due to an increase in σ_y^2 (Turnovsky, 1997, p. 442). Similarly, an increase in the variance of the foreign productivity shocks, $\sigma_{y^*}^2$, making domestic capital more attractive, tends to increase the growth rate if $\alpha > \alpha^*$. The positive effect on the growth rate is strengthened if $\gamma < 0$: consumption-wealth ratio falls due to an increase in $\sigma_{y^*}^2$.

Third, the impact of the public sector on the growth rate is analyzed. A higher size of the public sector, g, unambiguously increases, on the one hand, the growth rate since it raises the gross rate of return of the asset portfolio, ρ. On the other hand, consumption-wealth ratio can fall or raise, as we showed in section 3.3.1 above. Then, the overall effect on the growth rate of increasing the size of the public sector depends on which of the two effects (on ρ or C/W) dominate. For example, if consumption-wealth ratio falls or does not change, a higher size of the public sector enhances growth. Next, a higher variance of domestic public spending, σ_z^2, increases the growth rate of the economy for $\gamma < 0$, because consumption-wealth ratio falls (Turnovsky, 1997, p. 444). In contrast, we come to the opposite conclusions when either the covariance of domestic productivity and public spending shocks, σ_{yz}, or the covariance of foreign productivity shocks and public spending shocks, σ_{y^*z}, increases.

Fourth, in the case of a logarithmic utility function, the growth rate is

[16] See Turnovsky (1997, Ch. 11) for more details on the impact of changes in exogenous variables on the growth rate of wealth.

given by the expression

$$\psi_o = \rho - g - \beta.$$

Then, it is easy to show that, for instance, as long as the impact of a higher size of the public sector, g, on the gross rate of return of the asset portfolio, ρ, is higher than unity, increasing the size of the public sector is growth-enhancing.

Finally, we should note that most of the literature shows that the impact of a higher size of the public sector on growth depends basically on whether $\partial\rho/\partial g$ is higher than unity or not. However, our model shows that, once risk is introduced, we find a more complex relationship between the size of the public sector and the growth rate of wealth.

3.3.3 Welfare

Economic welfare is given by the value function used to solve the problem of intertemporal optimization, shown in equation (3.30). Thus, if we totally differentiate equation (3.30), we obtain, after some algebra, that

$$\frac{dV}{V} = (\gamma - 1)\frac{d(C/W)}{C/W},$$

where we observe that only changes in the optimal consumption-wealth ratio (influenced by the public sector, risk, and so on) have an impact on economic welfare. A higher optimal consumption-wealth ratio can improve or deteriorate the welfare of the domestic representative agent. Since C/W is positive in equation (3.30), the value function can take either positive or negative values, depending on the sign of the coefficient γ, subject to $\gamma V(W) > 0$. For the case $\gamma < 0$, anything that increases the optimal consumption-wealth ratio raises welfare. Thus, for instance, a higher size of the public sector, if it increases the optimal consumption-wealth ratio, generates higher welfare if $\gamma < 0$. However, we should note that increasing growth does not necessarily raise welfare.

3.3.4 Open versus closed economy

At this point it is convenient to obtain the difference between the variance of the growth rate in an open economy, shown in equation (3.23), and the same variance in a closed economy, shown in equation (3.27). The difference between both variances can be given, after some algebra, by

$$\sigma_{w,o}^2 - \sigma_{w,c}^2 = \Delta n_d^* \left(n_d^* - 2\widetilde{n}_d^*\right). \tag{3.36}$$

where

$$\widetilde{n}_d^* = \frac{\alpha^2\sigma_y^2 - \alpha\alpha^*\sigma_{yy^*} - \alpha\sigma_{yz} + \alpha^*\sigma_{y^*z}}{\Delta},$$

is the share of the domestic portfolio materialized in foreign capital that minimizes the variance of the growth rate given by equation (3.23).

First, if we substract equation (3.26) from equation (3.21) we find out after some algebra, via equation (3.36), that

$$\left(\frac{C}{W}\right)_o - \left(\frac{C}{W}\right)_c = -\frac{1}{1-\gamma}\left\{0.5\gamma\left(1-\gamma\right)\Delta n_d^{*^2}\right\}. \tag{3.37}$$

The difference between both consumption-wealth ratios depends critically upon the value of the parameter γ. Thus, if $\gamma < 0$ consumption-wealth ratio is higher in an open economy than in a closed economy, assuming an interior solution for the value of portfolio shares. An easy way to explain that result can be found focusing on the case $n_d = \widetilde{n}_d$, where

$$\widetilde{n}_d = 1 - \widetilde{n}_d^* = \frac{\alpha^{*^2}\sigma_{y^*}^2 - \alpha\alpha^*\sigma_{yy^*} + \alpha\sigma_{yz} - \alpha^*\sigma_{y^*z}}{\Delta},$$

is the share of the domestic portfolio materialized in domestic capital that minimizes the variance of the growth rate shown in equation (3.23). Then we obtain, from equation (3.36), that the variance of the growth rate in an open economy is lower than in a closed economy, $\sigma_{w,o}^2 < \sigma_{w,c}^2$. A reduction in the variance of the growth rate is equivalent to an increase in the gross rate of return of the asset portfolio. That, in turn, originates a negative substitution effect and a positive income effect on the consumption-wealth ratio. For instance, if $\gamma < 0$, income effect is stronger than substitution effect and consumption-wealth ratio in an open economy is higher than in a closed economy. In addition, the higher the value of the optimal share of the domestic portfolio materialized in foreign capital, n_d^*, the higher the

Table 3.2: Comparing growth rates

	$\gamma > 0$	$\gamma = 0$	$\gamma < 0$
$\alpha > \alpha^*$	$\psi_o \lesseqgtr \psi_c$	$\psi_o < \psi_c$	$\psi_o < \psi_c$
$\alpha = \alpha^*$	$\psi_o > \psi_c$	$\psi_o = \psi_c$	$\psi_o < \psi_c$
$\alpha < \alpha^*$	$\psi_o > \psi_c$	$\psi_o > \psi_c$	$\psi_o \lesseqgtr \psi_c$

difference between the results in an open economy and those in a closed economy.

Second, we can compare the growth rate in an open economy to a closed economy departing from equation (3.24) corresponding to an open economy and substracting from it that corresponding to a closed economy

$$\psi_o - \psi_c = n_d^*(\alpha^* - \alpha) - \left[\left(\frac{C}{W} \right)_o - \left(\frac{C}{W} \right)_c \right].$$

Thus, the growth rate in an open economy can be higher than, equal to, or lower than that in a closed economy, depending on the signs of two terms. For example, for $\gamma < 0$:

- If $\alpha \geq \alpha^*$, the growth rate of wealth in an open economy is lower than in a closed economy. Consumption-wealth ratio in an open economy is higher than in a closed economy and, additionally, if $\alpha \geq \alpha^*$, then the gross rate of return of the asset portfolio in an open economy is lower than or equal to the marginal physical product of the domestic capital.

- If $\alpha > \alpha^*$, then the growth rate of assets in an open economy can be higher than, equal to, or lower than that in a closed economy.

Table 3.2 sums up the comparison between the growth rate in an open economy to that in a closed economy.

Finally, focusing on welfare, since consumption-wealth ratio in an open economy should be higher than in a closed economy for $\gamma < 0$, as shown above in equation (3.37), welfare should be higher in a risky open economy than in a risky closed economy. Do note that welfare, which is given by the value function in (3.30), depends mainly on consumption-wealth ratio. This result adds insights to those shown in Obstfeld (1994) and Turnovsky (1997, Ch. 11), where they analyze the impact on welfare when changing from a domestic closed economy with low-yield and no risk (or relatively low risk) assets to an open economy with high-yield and high-risk assets, among other things. Obstfeld (1994, p. 1326-27) shows that "international risk-sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth is the attendant world portfolio shift from safe, but low-yield, capital into riskier, high-yield capital". In addition, Turnovsky (1997, p. 439) shows that for a logarithmic utility function "the higher growth rate more than offsets the additional risk, and the opportunity to invest in a higher return, higher risk foreign asset improves welfare". However, we should note that our conclusion is not based on low risk-high risk considerations, but on closed economy-open economy considerations, provided that the size of the public sector is exogenously given. In addition, we should point out that this result depends exclusively on the sign of the parameter γ.

3.4 The optimal size of the public sector

Now, we turn to the size of the public sector g that maximizes welfare or, for short, the optimal size of the public sector. A crucial characteristic of the model is that domestic productive government expenditure generates an externality on the foreign economy, and viceversa. That leads us to consider two different scenarios in an open economy. In the first scenario, we assume that the domestic public sector only takes into account the impact of public spending on the domestic economy and not that impinged on the foreign economy: the domestic productive public sector does not internalize the externality. Thus, we obtain a unilateral (or one-sided) optimal size of the public sector, which is perceived individually as optimal, but it is not so for the world as a whole. In the second scenario, the domestic public sector is assumed to take into account the impact of public spending on both domestic and foreign economies, that is, the domestic public sector internalizes the externality. Thus, we obtain a harmonized size of the public sector which is optimal for the world as a whole.

In addition, we obtain the optimal size for a domestic closed economy. Next, we discuss whether the size of the public sector that maximizes welfare coincides with that which maximizes growth. Finally, we analyze whether more open economies are associated to a higher optimal size of the public sector, first on the more simple case where public spending only influences productivity, but not volatility, and later, on the more general case. For simplicity, in this section we assume that $\sigma_{yy^*} = \sigma_{yz} = \sigma_{y^*z} = \sigma_{yz^*} = \sigma_{y^*z^*} = 0$.

3.4.1 Open economy: the unilateral optimal size

In the first scenario in an open economy we assume that the domestic public sector takes into account the impact of public spending only on the domestic economy. Thus, the domestic public sector does not internalize the externality caused in the foreign economy. To obtain the unilateral optimal size of the public sector the expression in the right hand side of the Bellman equation (3.14) is partially differentiated with respect to g, and then substituting the value function (3.30) into the result obtained, we find that

$$\left(\delta - \theta \hat{g}_{o,u}\right) n_d - (1-\gamma)\left(\delta - \theta \hat{g}_{o,u}\right) n_d^2 \sigma_y^2 - 1 = 0, \tag{3.38}$$

where $\hat{g}_{o,u}$ denotes the unilateral optimal size of the public sector. Equation (3.38) means that, at the optimal size, the marginal return of an additional unit of public spending, $\left(\delta - \theta \hat{g}_{o,u}\right) n_d - (1-\gamma)\left(\delta - \theta \hat{g}_{o,u}\right) n_d^2 \sigma_y^2$, must be equal to the marginal cost, 1. The marginal return of public spending includes, in turn, the productive effect, $\left(\delta - \theta \hat{g}_{o,u}\right) n_d$, plus the volatility effect, $-(1-\gamma)\left(\delta - \theta \hat{g}_{o,u}\right) n_d^2 \sigma_y^2$ in the domestic economy only.[17]

From equation (3.38) the unilateral optimal size of the public sector in an open economy is implicitly derived as

$$\hat{g}_{o,u} = \frac{n_d \delta \left[1 - (1-\gamma) n_d \sigma_y^2\right] - 1}{n_d \theta \left[1 - (1-\gamma) n_d \sigma_y^2\right]}, \tag{3.39}$$

where both productive and volatility effects determine the optimal size of the public sector. The terms in the numerator capture the first order effects on growth and volatility, and the terms in the denominator the second order effects.

[17] Do note that we assumed above that $g < \delta/\theta < 1$. In addition, for the unilateral optimal size of the public sector $\hat{g}_{o,u}$ to be positive, the marginal return derived from public spending is required to be higher than its marginal cost for $\hat{g}_{o,u} = 0$, that is, $n_d \delta \left[1 - (1-\gamma)\overline{\alpha} n_d \sigma_y^2\right] > 1$. Furthermore, the second order condition for g, that is, concavity with respect to g, requires that $1 - (1-\gamma)\alpha n_d \sigma_y^2 > 0$.

3.4.2 Open economy: the harmonized optimal size

In the second scenario in an open economy we begin by assuming that the external effect of domestic public spending is internalized. The preferences of the central planner are represented by the sum of two isoelastic intertemporal utility functions, depending on domestic and foreign consumption, and both having equal weight

$$E_0 \int_0^{\infty} \left(\frac{1}{\gamma} C^{\gamma} e^{-\beta t} + \frac{1}{\gamma^*} C^{\gamma^*} e^{-\beta^* t} \right) dt; -\infty < \gamma, \gamma^* < 1. \tag{3.40}$$

The dynamics of domestic wealth are given by equations (3.10), (3.11) and (3.12). Foreign wealth, in turn, evolves according to equations (3.33), (3.34), and (3.35).

The objective of the central planner would consist in choosing the sizes of the public sectors, g and g^*, that maximize (3.40), subject to $W(0) = W_0$, $W^*(0) = W_0^*$, (3.10), (3.11), (3.12), (3.33), (3.34), and (3.35).

We introduce a value function, $G(W, W^*)$, which is defined as

$$\begin{aligned} G(W, W^*) = V(W) + V^*(W^*) = \\ Max_{\{g,g^*\}} \;\; E_0 \int_0^{\infty} \left(\frac{1}{\gamma} C^{\gamma} e^{-\beta t} + \frac{1}{\gamma^*} C^{\gamma^*} e^{-\beta^* t} \right) dt, \end{aligned} \tag{3.41}$$

subject to restrictions (3.10), (3.11), (3.12), (3.33), (3.34), (3.35), and given initial wealth.

The value function, given by equation (3.41), must satisfy the Bellman equation

$$\begin{aligned} \beta V(W) + \beta^* V^*(W^*) = Max_{\{g,g^*\}} \;\; & \left[\frac{1}{\gamma} C^{\gamma} + \frac{1}{\gamma^*} C^{\gamma^*} + V'(W) W \psi \right. \\ & \left. + 0.5 V''(W) W^2 \sigma_w^2 + V^{*'}(W^*) W^* \psi^* + 0.5 V^{*''}(W^*) W^{*^2} \sigma_{w^*}^2 \right]. \end{aligned} \tag{3.42}$$

Now, we focus on the case where both economies grow at the same rate, or, in a broader sense, where the rates of growth of both economies do not differ very much. That has been, in fact, the path traditionally followed in the literature on two-country endogenous growth models. In addition to tractability reasons, the literature has emphasized that if both economies did not grow at the same rate, one would become infinitely bigger compared to the other (Razin and Yuen, 1993; Lejour and Verbon, 1996). Thus, if we restrict to the case where consumption-wealth ratio, portfolio shares and the size of the public sector are the same in both countries, should domestic wealth and foreign wealth should grow at the same rate and we obtain a clear-cut solution.[18] Before going ahead, we should note that assuming that both economies grow at the same rate implies that the external effect of the domestic economy on the foreign economy is equal to the external effect of the foreign economy on the domestic economy or, alternatively, since the public sector budget is balanced, that tax revenues on the domestic economy paid by foreigners are equal to tax revenues on the foreign economy paid by domestic residents.

The right hand side of equation (3.42) is partially differentiated with respect to the harmonized optimal size of the public sector, g, in order to obtain the first order condition of this optimization. Plugging the value function (3.30) into the result obtained, we have that

$$\begin{aligned} &(\delta - \theta\widehat{g}_{o,h})\, n_d \left[1 - (1-\gamma)\, n_d \alpha \sigma_y^2\right] \\ &+ (\delta^* - \theta^*\widehat{g}_{o,h})\, n_d^* \left[1 - (1-\gamma)\, n_d^* \alpha^* \sigma_{y^*}^2\right] - 1 = 0, \end{aligned} \tag{3.43}$$

where $\widehat{g}_{o,h}$ denotes the harmonized optimal size of the public sector. Equation (3.43) shows that, at the optimal size, the marginal return of public spending,

$$(\delta - \theta\widehat{g}_{o,h})\, n_d \left[1 - (1-\gamma)\, n_d \alpha \sigma_y^2\right] + (\delta^* - \theta^*\widehat{g}_{o,h})\, n_d^* \left[1 - (1-\gamma)\, n_d^* \alpha^* \sigma_{y^*}^2\right],$$

must equalize the marginal cost, 1. The marginal return of public spending, in turn, reflects the impact of spending on the domestic economy, $(\delta - \theta\widehat{g}_{o,h})\, n_d \left[1 - (1-\gamma)\, n_d \alpha \sigma_y^2\right]$, plus the impact on the foreign economy, $(\delta^* - \theta^*\widehat{g}_{o,h})\, n_d^* \left[1 - (1-\gamma)\, n_d^* \alpha^* \sigma_{y^*}^2\right]$, both in terms of productivity and volatility.[19] Thus we obtain from equation (3.43) that the harmonized optimal size of the public sector in an open economy is implicitly given by

[18] Formally, this means that $n_d = n_f$, $n_d^* = n_f^*$, $\frac{C}{W} = \frac{C^*}{W^*}$, $g = g^*$, $dz = dz^*$, $\gamma = \gamma^*$, and $\beta = \beta^*$.

[19] That the harmonized optimal size of the public sector $\hat{g}_{o,h}$ is positive requires that the marginal return derived from public spending be higher than its marginal cost for $\hat{g}_{o,h} = 0$. This implies that $n_d \delta \left[1 - (1-\gamma)\, \overline{\alpha} n_d \sigma_y^2\right] + n_d^* \delta^* \left[1 - (1-\gamma)\, \overline{\alpha}^* n_d^* \sigma_{y^*}^2\right] > 1$. The second order condition for g requires that $1 - (1-\gamma)\, \alpha n_d \sigma_y^2 > 0$ and $1 - (1-\gamma)\, \alpha^* n_d^* \sigma_{y^*}^2 > 0$.

$$\hat{g}_{o,h} = \frac{n_d \delta \left[1 - (1-\gamma)\alpha n_d \sigma_y^2\right] + n_d^* \delta^* \left[1 - (1-\gamma) n_d^* \sigma_{y^*}^2\right] - 1}{n_d \theta \left[1 - (1-\gamma)\theta n_d \sigma_y^2\right] + n_d^* \theta^* \left[1 - (1-\gamma) n_d^* \sigma_{y^*}^2\right]}. \tag{3.44}$$

The terms in the numerator capture the first order effects on growth and volatility, weighted by the appropriate portfolio shares, whereas the terms in the denominator reflect second order effects.

3.4.3 Closed economy

Turning to the domestic closed economy, we could obtain the optimal size of the public sector, $\hat{g}_c$, solving the corresponding problem for the domestic representative agent by setting $n_d = 1$ in any of the above equations (3.38) or (3.43)

$$(\delta - \theta\widehat{g}_c) - (1-\gamma)(\delta - \theta\widehat{g}_c)\alpha\sigma_y^2 - 1 = 0, \tag{3.45}$$

so that the optimal size of the public sector in a closed economy is implicitly given by

$$\hat{g}_c = \frac{\delta\left[1 - (1-\gamma)\alpha\sigma_y^2\right] - 1}{\theta\left[1 - (1-\gamma)\alpha\sigma_y^2\right]}. \tag{3.46}$$

Do note that the optimal size of the public sector in a foreign closed economy can be found, in turn, by setting $n_d = 0$ in equation (3.44).

Thus, in a risk-free closed economy, the optimal size of the public sector is given by

$$\hat{g}_c = \frac{\delta - 1}{\theta}. \tag{3.47}$$

The mathematical result in (3.47) cannot be compared to that found in Barro (1990): the optimal size of the public sector in Barro (1990) is equal to the exponent on public spending in a Cobb-Douglas production function that exhibits constant returns to scale.[20] Instead, our approach to incorporate

[20] There are additional terms in the optimal size of the public sector shown in Turnovsky (1998, 1999).

public spending has been different as we showed in Section 3.2.1, following Gallaway and Vedder (1995) and Vedder and Gallaway (1998). However, the important point is that we can compare the results obtained in equations (3.39) and (3.44) to benchmark results in equations (3.46) or (3.47) in our model. Therefore, we can *compare* the *conclusions* derived from our model to those in the core literature, even though we *cannot compare* the *mathematical results* in equations (3.39), (3.44), (3.46), and (3.47) of our model vis-à-vis those in the core literature.

We find that the optimal size of the public sector in a closed economy with risk, given by (3.46), is lower than that with no risk, given by (3.47). This result was already shown in Turnovsky (1999, p. 899). However, his additional result that if there was no production risk in the domestic economy "the optimal size of the productive government in the stochastic open economy coincides with that in the deterministic closed economy" no longer applies. Thus, the harmonized optimal size of the public sector in a stochastic open economy, given by equation (3.44), is not necessarily equal to the optimal size of the public sector in a deterministic closed economy with no domestic production risk [equation (3.47)].

3.4.4 Growth vs. welfare maximizing

Now, we can compare the optimal size of the public sector to the size that maximizes growth, where the growth rate of assets is shown in equation (3.24). Partially differentiating equation (3.24) with respect to g, we derive the unilateral size of the public sector that maximizes the growth rate as

$$\overline{g}_{o,u} = \frac{n_d\delta\left[1-\gamma\left(1-\gamma\right)n_d\sigma_y^2\right]-1}{n_d\theta\left[1-\gamma(1-\gamma)n_d\sigma_y^2\right]}. \tag{3.48}$$

If we substract equation (3.39) from equation (3.48), it can be easily shown, after some algebra, that the unilateral size of the public sector that maximizes the growth rate, $\overline{g}_{o,u}$, is unambiguously higher than the unilateral optimal size of the public sector, $\widehat{g}_{o,u}$. That has been shown already in Turnovsky (1998, p. 16): "The intuition is that the maximization of the growth rate entails more risk than the risk averse agent, concerned with his time profile of consumption, finds to be optimal". The result that welfare maximizing and growth maximizing objectives do not amount to the same thing has already been derived in other contexts, such as models where productive public spending influences adjustment costs of new investment, where the productive good provided by the public sector is introduced as a stock, instead of as a flow [see Turnovsky (2003, p. 20) for more details and references] or where the representative agent is not risk averse (Turnovsky, 1998, p. 16). In addition, we should note that if, in opposition to our model, public spending is regarded to be volatility-reducing (besides productivity-enhancing), we shall conclude that the unilateral optimal size of the public sector, $\widehat{g}_{o,u}$, is higher than that which maximizes growth, $\overline{g}_{o,u}$. Instead, if there is no risk, both maximizing welfare and growth are equivalent in the case of Cobb-Douglas production function (Barro, 1990, pp. S111-S112)[21]. In addition, the results obtained for the unilateral optimal size can be easily extended to the harmonized optimal size and to the optimal size in a risky closed economy.

3.4.5 Open versus closed economy

Here we discuss whether higher values of foreign capital holdings out of domestic wealth, n_d^*, that is, more open economies, are associated to a higher optimal size of the public sector (Turnovsky, 1999).

A digression: productive-only spending

Now, only the case where public spending influences productivity is analyzed. The results of the model become much simpler. If the impact of public spending on volatility is null, the domestic production function in equation (3.4) becomes

$$dY = \alpha K dt + \overline{\alpha} K dy,$$

where α was given by equation (3.5) and it is affected by the size of the public sector. Instead, the parameter $\overline{\alpha}$ is a constant and it is not influenced by the size of the public sector. It is easy to show that the optimal size of the public sector in a domestic closed economy is equal to that where there

[21] However, if the production function were not Cobb-Douglas "the relative size of government that maximizes utility turns out to exceed the value that maximizes the growth rate [...] if and only if the magnitude of the elasticity of substitution between g [the quantity of public services provided to each household-producer] and k [capital per worker] is greater than unity" (Barro, 1990, p. S112).

is no risk, given by equation (3.47). Symmetrically, the optimal size of the public sector in a foreign closed economy, $\hat{g}_c^*$, is given by

$$\hat{g}_c^* = \frac{\delta^* - 1}{\theta^*}. \tag{3.49}$$

We can first establish going back to equation (3.39), that the unilateral optimal size of the public sector is given by

$$\hat{g}_{o,u} = \frac{\delta n_d - 1}{\theta n_d}.$$

It can be easily checked that the optimal size of the public sector in a closed economy, $\hat{g}_c$, is unambiguously higher than the unilateral optimal size of the public sector, $g_{o,u}$, for interior values of portfolio shares. The reason behind is that the higher the value of the portfolio share n_d^* is, the lower the level of internalization of the externality will be. The domestic public sector finds optimal to reduce the unilateral optimal size since the benefit of public spending for the domestic public sector becomes lower. In addition, the more open the domestic economy is, that is, the higher the value of the portfolio share n_d^*, the lower the unilateral optimal size of the public sector will be.

Second, we can establish following equation (3.46) that the harmonized optimal size of the public sector is given by

$$\widehat{g}_{o,h} = \frac{\delta n_d + \delta^* n_d^* - 1}{\theta n_d + \theta^* n_d^*},$$

which implies that the harmonized optimal size of the public sector in an open economy is always between the values of the optimal size of the public sector in both closed economies. That is not surprising since the world economy is a closed economy after all. Then it can be easily shown for the harmonized optimal size that

$$sgn\left(\widehat{g}_{o,h} - \widehat{g}_c\right) = sgn\left(\widehat{g}_c^* - \widehat{g}_c\right).$$

Thus, for example, we find that the harmonized optimal size of the public sector, $\widehat{g}_{o,h}$, is higher than the optimal size in domestic closed economy, $\widehat{g}_c$, if and only if $\widehat{g}_c^* > \widehat{g}_c$, that is, the optimal size of the public sector in a foreign closed economy is higher than in a domestic closed economy. The reason behind is that in that case the marginal impact of public spending is higher in a foreign economy.

The general case

Now we turn to the more general case where public spending influences volatility as well as productivity. To begin with, the first order condition for the unilateral optimal size of the public sector in an open economy $\widehat{g}_{o,u}$ is given by equation (3.38), whereas for the case of a closed economy, $\widehat{g}_c$, is given by equation (3.45). Comparing equations (3.38) and (3.45) we observe that, on the one hand, the impact of public spending on productivity, $(\delta - \theta g)\, n_d$, is lower in an open economy than in a closed economy, $\delta - \theta g$, for the same size of the public sector, g, as we showed above in the productive-only case. On the other hand, the influence of public spending on volatility is higher (that is, less negative) in an open economy, $-(1-\gamma)(\delta - \theta g)\,\alpha n_d^2 \sigma_y^2$, than in a closed economy, $-(1-\gamma)(\delta - \theta g)\,\alpha \sigma_y^2$, for the same size g, since the impact in an open economy depends on the share of the portfolio materialized in domestic capital, n_d. Thus the net impact depends upon which of the effects dominate. For example, obtaining that the unilateral optimal size of the public sector in an open economy $\widehat{g}_{o,u}$ is higher than that in a closed economy $\widehat{g}_c$ implies that if we introduce $\widehat{g}_{o,u}$ in equation (3.45) for the first order condition of the optimal size in a closed economy then we should have that

$$(\delta - \theta \widehat{g}_{o,u}) - (1-\gamma)(\delta - \theta \widehat{g}_{o,u})\,\alpha \sigma_y^2 < 1, \tag{3.50}$$

due to the second order condition required for the size of the public sector g to be a maximum. Combining both equations (3.38) and (3.50), and after some algebra, we get that the unilateral optimal size will be higher than the optimal size in a closed economy if and only if

$$(1-\gamma)(\delta - \theta \widehat{g}_{o,u})(1+n_d)\,\alpha \sigma_y^2 > (\delta - \theta \widehat{g}_{o,u}).$$

Second, the harmonized optimal size of the public sector is given by equation (3.43). If we compare equation (3.43) with equation (3.45) we can easily show that, on the one hand, the impact of public spending on productivity in an open economy, $(\delta - \theta g)\, n_d + (\delta^* - \theta^* g)\, n_d^*$, compared to that in a closed economy, $\delta - \theta g$, for the same size g, depends on the difference between the optimal size of the public sector in a foreign closed economy $\widehat{g}_c^*$ shown in (3.49) and that in a domestic closed economy $\widehat{g}_c$ given by (3.47), as we showed above in section 3.4.5. For instance, if $\widehat{g}_c^* > \widehat{g}_c$ then the impact of public spending on productivity is higher in an open economy than in a closed economy. On the other hand, the impact of public spending on volatility in an open economy,

$$-\left(\delta-\theta g\right)\left(1-\gamma\right)\alpha n_d^2\sigma_y^2-\left(\delta^*-\theta^* g\right)\left(1-\gamma\right)\alpha^* n_d^{*^2}\sigma_{y^*}^2,$$

in relation to that in a closed economy, $-\left(\delta-\theta g\right)\left(1-\gamma\right)\alpha\sigma_y^2$, depends mainly on the variances of productivity shocks in both economies and the values of the portfolio shares n_d and n_d^*. For example, obtaining that the harmonized optimal size of the public sector $\widehat{g}_{o,h}$ is higher than that in a closed economy $\widehat{g}_c$ implies that if substitute $\widehat{g}_{o,h}$ into equation (3.45) for the first order condition for the optimal size in a domestic closed economy

$$\left(\delta-\theta\widehat{g}_{o,h}\right)-\left(1-\gamma\right)\left(\delta-\theta\widehat{g}_{o,h}\right)\alpha\sigma_y^2<1. \tag{3.51}$$

so that the second order condition for the optimal size g is satisfied. If we combine equation (3.43) with equation (3.51), then we obtain, after some algebra, the result that $\widehat{g}_{o,h}>\widehat{g}_c$ provided that

$$\begin{aligned}&\left(\delta^*-\theta^*\widehat{g}_{o,h}\right)+\left(1-\gamma\right)\left(\delta-\theta\widehat{g}_{o,h}\right)\left(1+n_d\right)\alpha\sigma_y^2> \\ &\qquad\left(\delta-\theta\widehat{g}_{o,h}\right)+\left(1-\gamma\right)\left(\delta^*-\theta^*\widehat{g}_{o,h}\right)n_d^*\alpha^*\sigma_{y^*}^2.\end{aligned} \tag{3.52}$$

In equation (3.52) we first observe the result obtained above in section 3.4.5 for productive-only spending: the harmonized optimal size of the public sector, $\widehat{g}_{o,h}$, will be higher than that in a domestic closed economy, $\widehat{g}_c$, if and only if the optimal size of the public sector in a foreign closed economy $\widehat{g}_c^*$ is higher than that in a domestic closed economy $\widehat{g}_c$. In addition, we have that the impact of public spending on volatility in an open economy can be higher or lower than that in a closed economy. However, under most conditions the impact will be higher (that is, less negative) in an open economy. For example, if we focus, for simplicity, on the case where the marginal product in both countries is equal, that is, $\alpha=\alpha^*$, the marginal impact of public spending on productivity is equal in both economies, that is, $\delta-\theta\widehat{g}_{o,h}=\delta^*-\theta^*\widehat{g}_{o,h}$, and the variances of productivity shocks are similar, we know from section 3.3.4 that the variance of the growth rate in an open economy is lower than that in a closed economy, $\sigma_{w,o}^2<\sigma_{w,c}^2$, as given by equation (3.36). In addition, since the impact on volatility is the only factor that can originate a difference between the optimal size in an open economy and that in a closed economy, then we conclude that the harmonized optimal size of the public sector in an open economy is higher than in a closed economy because the impact on volatility is higher (that is, less negative) in an open economy than in a closed economy.

These results offer additional insights to those of Turnovsky (1999), where a more open economy is unambiguously associated with a higher optimal size in an open economy, provided that the domestic economy holds positive stocks of foreign capital in a small open economy. Here we have found two reasons why the harmonized optimal size of the public sector in an open economy should be higher than that in a closed economy. The first one is based on the fact that the optimal size of the public sector in a foreign closed economy $\widehat{g}_c^*$ can be higher than that in a domestic closed economy $\widehat{g}_c$. That depends upon the difference in the marginal impact of public spending on productivity in both countries. The second reason has to do with risk diversification. Since the impact of public spending on volatility is surely higher (that is, less negative) in an open economy than in a closed economy, the harmonized optimal size of the public sector in an open economy should be higher than in a closed economy. Turnovsky (1999, p. 889) bases his results on "the country's ability to export its domestic risk, rather than due to insulating the country from foreign risk, as argued by Rodrik [1998]". We should note that our argument based on the higher impact of public spending on volatility due to risk diversification argument resembles more Rodrik's (1998, p. 1011) "insulation function" than Turnovsky's "risk exporting" argument. However, Rodrik emphasizes the central role played by the public sector in insulating against external risk, whereas here the result is the consequence of the risk diversification achieved through perfect capital mobility.

3.5 Conclusions

The impact of productive public spending and risk on long run growth is an important issue for economic policy. However, the analysis has been mostly relegated either to a closed economy or an small open economy. This paper extends a two-country stochastic AK growth model based on Turnovsky (1997, Ch. 11) incorporating a production good that enhances both the productivity of physical marginal product of private capital and volatility [Barro (1990) for the original deterministic model and Turnovsky (1998, 1999) for an extension to a risky closed economy and a small open economy, respectively]. The conclusions are summarized in seven categories.

First, having obtained the world equilibrium, we have reviewed how consumption-wealth ratio, the growth rate of assets and welfare respond to changes in exogenous variables, provided that the size of the public sector is exogenously given. Most of the results are familiar. However, we have shown that a higher size of the public sector, enhancing productivity and volatility, implies a richer analysis of the impact on the consumption-wealth ratio and the growth rate. For example, if the net impact of a higher size of the public sector is positive, consumption-wealth should raise. Since welfare depends basically on consumption-wealth ratio, public spending influences welfare altering consumption-wealth ratio.

Second, we have compared the behavior of key economic variables in an open economy in contrast to a closed economy. Since an open economy can achieve a lower variance of the growth rate through risk diversification, consumption-wealth ratio should be higher in an open economy than in a closed economy, assuming that the size of the public sector is exogenously given. Next, we have shown that the growth rate in an open economy is lower than in a closed economy if the marginal physical product of domestic capital is higher than that of foreign capital. Additionally we should note that welfare should be higher in an open economy than in a closed economy, since welfare depends upon consumption-wealth ratio.

Third, we have obtained the optimal size of the public sector in an open economy. Since domestic productive government expenditure generates an externality on the foreign economy in the model we have considered two different scenarios for an open economy. In the first scenario we assume that the domestic productive public sector only takes into account the impact of productive government spending on the domestic economy and not that impinged on the foreign economy. In the second scenario we assume that the domestic productive public sector takes into account the impact of productive government spending on both domestic and foreign economies. We should note that obtaining a closed form harmonized optimal size requires assuming that both economies grow exactly at the same rate. It has been derived that the optimal size of the public sector in a closed economy with risk is lower than that with no risk, as is Turnovsky (1999). However, we have shown that in case there is no domestic production risk the harmonized optimal size of the public sector in an open economy does not have to be necessarily equal to the optimal size in a domestic closed economy, in contrast to Turnovsky (1999). Finally, we found out that the size that maximizes welfare is lower than that which maximizes growth due to risk aversion, as in Turnovsky (1998).

Fourth, the optimal size of the public sector in an open economy has been compared to that in a closed economy. In the case public spending is productive-only, we find that the unilateral optimal size of the public sector in an open economy should be unambiguously lower than in a closed economy since public spending is not fully internalized. In contrast, we have concluded that the harmonized optimal size of the public sector should be higher than in a closed economy provided that the optimal size of the public in a foreign closed economy is higher than in a domestic closed economy. The marginal impact of public spending on productivity is higher abroad than at home. In the more general case where public spending influences volatility as well, we have concluded that the unilateral optimal size of the public sector will be higher than in a domestic closed economy if and only if the marginal impact on volatility is higher (less negative) than on productivity. As regards the harmonized case, we have argued that there are two channels through which the harmonized optimal size of the public sector should be higher than in a domestic closed economy. The first channel is that the optimal size of the public sector in a foreign closed economy should be higher than in a domestic closed economy, as argued in the productive-only case. The second channel has to do with the higher positive impact (that is, less negative) of public spending on volatility in an open economy than in a closed economy due to risk diversification. We have found that the second channel goes along the same lines as the argument in Rodrik (1998) about insulating an open economy from external risk through the intervention of the public sector rather than that in Turnovsky (1999) about the ability to export domestic risk. However, Rodrik attributes a central role to the public sector, while here our argument is based on the relatively lower impact of public spending on volatility achieved by the risk diversification in a world with perfect capital mobility.

Fifth, we should note that this model can easily be extended so that public spending, instead of being volatility-enhancing, is volatility-reducing. That would reverse most of the conclusions of this paper. For example, it can be easily shown that if public spending is volatility-reducing as well as productivity-enhancing then the size of the public sector that maximizes welfare should be higher than that which maximizes growth. Further, it implies significative changes as regards the conclusions about the optimal size of the public sector in an open economy as compared to those in a closed economy.

Sixth, we must note that the model has important limitations. We have analyzed the equilibrium in a world economy where, even though the domestic and the foreign economies are different, the representative agents are identical, and the equilibrium is characterized by identical balanced growth rates. Therefore, it is not suitable to study structural differences between countries, but it could be useful for countries where their rates of growth are quite similar.

Finally, we should point out possible paths for future research. We could relax the assumption of continuous budget equilibrium and introduce public bonds in the model. However, that would enormously increase its complexity. Introducing money is also an interesting element that could be integrated into a two-country world economy. The incorporation of congestion in the provision of the production good would be another possibility to extend the model. The inclusion of more complex strategic interaction would be an additional interesting feature.

Chapter 4

The current account and the new rule in a two-country world

4.1 Introduction

Recent studies [Kraay and Ventura (KV hereafter) (2000), for example][1] propose incorporating new features to the intertemporal approach to the current account, so that the theory fits the empirical evidence on current accounts more satisfactorily. According to the standard version of the intertemporal approach[2], which KV have termed "the traditional rule", the impact of transitory income shocks (fluctuations in output, for example) on the current account is equal to the amount of saving generated by transitory income shocks in all countries. The reason behind the result lies in the fact that, in addition to saving part of the transitory income shock, so that consumption is smoothed, "in existing intertemporal models of the current account, countries invest the marginal unit of wealth in foreign assets" (KV, 2000, p. 1138). However, the empirical evidence seems to be at odds with the theory. In fact, according to Ventura (2003, p. 510), "there are some patterns in the current accounts of industrial countries that are inconsistent with the basic theory that international economists have been using for more than two decades".

[1] Other references include Ventura (2001), Kraay and Ventura (2002), and Ventura (2003).

[2] See Obstfeld and Rogoff (1995, 1996), Razin (1995), and Frenkel, Razin and Yuen (1996), for example, for excellent surveys on the intertemporal approach to the current account.

KV (2000, p. 1138) have had a remarkable insight challenging the traditional rule by postulating "that the country invests the marginal unit of wealth as the average one". Then they obtain that "the current account response is equal to the saving generated by the shock multiplied by the country's share of foreign assets in total assets" (p. 1137), which they have termed "the new rule". The new rule implies a response which depends on the net foreign asset (creditor or debtor) position of the country, thus departing in a superbly simple way from the traditional rule. In addition, the empirical evidence seems to support the new rule. Therefore, KV provide a new framework that coherently relates the theory on the intertemporal approach to the current account and the evidence on current accounts. However, the characterization of the new rule is based on a small open economy model and therefore it does not focus on important channels through which the foreign economy influences on the domestic economy.

This paper analyzes the impact of transitory income shocks on the current account by extending the new rule to a two-country stochastic AK growth model. We begin by reviewing the traditional rule and the new rule. Then an extension of the new rule to a two-country world is characterized. We briefly discuss the sample data on which the testing of the model is based, since we use the same data as KV: thirteen OECD countries for the 1973-1995 period. Next, the empirical relevance of the extended new rule in contrast to the other rules, either the traditional rule or the new rule, is studied. Finally, we conclude indicating possible avenues for future research.

4.2 Theory

4.2.1 Basic structure

The world economy is composed of two countries, each of them producing only one homogeneous good. In each country there exists a representative agent with an infinite time horizon. The homogeneous good produced by both countries can be either consumed or invested in capital without having to incur in any kind of adjustment costs. There are three assets: domestic capital, foreign capital and bonds. Unstarred variables refer to the domestic economy, whereas the starred variables refer to the foreign economy. Both domestic capital, K, and foreign capital, K^*, can be owned by the domestic representative agent or the foreign representative agent. The subscript d

denotes the holdings of assets of the domestic representative agent and the subscript f denotes the holdings of assets of the foreign representative agent. So it must be satisfied that

$$\begin{aligned} K &= K_d + K_f \\ K^* &= K_d^* + K_f^*. \end{aligned}$$

Domestic production is obtained using only domestic capital, K, through an AK function, and it is expressed through a first order stochastic differential equation, so that production flow dY is subject to a stochastic disturbance

$$dY = \alpha K dt + \alpha K dy,$$

where $\alpha > 0$ is the (constant) marginal physical product of capital and dy represents a proportional domestic productivity shock. More precisely, dy is the increment of a stochastic process y. Those increments are temporally independent and are normally distributed. They satisfy that $E(dy) = 0$ and $E(dy^2) = \sigma_y^2 dt$.[3] We omit, for convenience, formal references to time, although those variables depend on time. We must note that dY indicates the flow of production, instead of Y, as is ordinarily done in stochastic calculus. We should note that here the marginal product of physical capital is constant for simplicity, whereas in KV it is not.

The foreign economy is structured symmetrically to the domestic economy. Thus, foreign production is carried out using capital domiciled abroad, K^*, with a production function very similar to the one in the domestic economy

$$dY^* = \alpha^* K^* dt + \alpha^* K^* dy^*,$$

where $\alpha^* > 0$ is the marginal physical product of capital and dy^* represents a proportional foreign productivity shock. The term dy^* is the increment of a stochastic process y^*. Those increments are temporally independent and are distributed normally, satisfying that $E(dy^*) = 0$ and that $E(dy^{*^2}) = \sigma_{y^*}^2 dt$.

[3] That is, the production flow follows a Brownian motion with drift αK and with variance $\alpha^2 K^2 \sigma_y^2$.

In addition, bonds pay an exogenous instantaneous risk-free rate of interest η. The domestic economy can be lending to (and thus $B > 0$) or borrowing from (and thus $B < 0$) the foreign economy.[4] Thus B denotes the net position of risk-free loans. The wealth of the domestic representative agent, W, and the wealth of the foreign representative agent, W^*, therefore will be

$$W = K_d + K_d^* + B \tag{4.1}$$
$$W^* = K_f + K_f^* - B. \tag{4.2}$$

The net foreign asset position for the domestic economy, P, is defined as

$$P = K_d^* - K_f + B, \tag{4.3}$$

where changes in any of those variables lead to changes in the net foreign asset position.

The current account of the domestic economy, CA, is defined as the variation in its net foreign asset position given by (4.3), dP. Thus we have that

$$CA = dP = dK_d^* - dK_f + dB. \tag{4.4}$$

This means that, for example, the current account is positive if the variation in K_d^* and B is higher than the variation in K_f.

We can convert equation (4.4), after a bit of algebra, into

$$CA = dW - dK = dW - dW\frac{dK_d}{dW} - dW^*\frac{dK_f}{dW^*}. \tag{4.5}$$

Thus equation (4.5) is the national account identity, where the current account balance is equal to the variation in domestic wealth minus the variation in domestic capital. Please note that the variation in domestic wealth, dW, is equal to the national savings for the period, S, that is, national income minus (private and public) consumption. Additionally, the variation in domestic capital, dK, is equal to the domestic investment for the period.

[4]Since the world economy is composed of the domestic and foreign economies, the assumption of an exogenous risk-free rate of interest is "heroic" indeed. However, since the purpose of this chapter is not to analyze the world macroeconomic equilibrium as such, but to focus on the impact of shocks on the current account, we believe that assumption is not so heroic. Similar assumption is made in Obstfeld (1994) in a different context.

4.2.2 The traditional rule

Since "in existing intertemporal models of the current account countries invest the marginal unit of wealth in foreign assets" (KV, p. 1138), then $dK_d/dW = 0$. Additionally, the small open economy assumption implies that $dK_f = 0$.[5] Therefore, the current account, via equation (4.5), becomes

$$CA = dW.$$

Thus "[...] these models predict that favorable transitory income shocks generate current account responses that are equal to the saving generated by the shock. [...] it implies that all countries respond to transitory income shocks with surpluses in the current account" (KV, p. 1138). KV term this idea the traditional rule.

4.2.3 The new rule and the extended new rule

KV depart from the standard approach following other assumption on how countries save and invest a transitory income shock. The preferences of the domestic representative agent are represented by an isoelastic intertemporal utility function where she obtains utility from consumption, C

$$E_0 \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\beta t} dt \qquad (4.6)$$
$$-\infty < \gamma < 1.$$

The welfare of the domestic representative agent in period 0 is the expected value of the discounted sum of instantaneous utilities, conditioned on the set of disposable information in period 0. The parameter β is a positive subjective discount rate (or rate of time preference). The restrictions on the utility function are necessary to ensure concavity with respect to consumption.

The domestic representative agent consumes at a deterministic rate $C(t)dt$ in the instant dt and thus the dynamic budget restriction can be expressed in the following way

$$dW = [\alpha K_d + \alpha^* K_d^* + \eta B]\, dt + [\alpha K_d dy + \alpha^* K_d^* dy^*] - C dt. \qquad (4.7)$$

[5] "In keeping with the small country assumption, we have implicitly assumed that foreign holdings of domestic capital are constant" in KV (p. 1145, footnote 7).

If we define the following variables for the domestic representative agent

$$\begin{aligned}
n_d &\equiv \frac{K_d}{W} = \text{ share of the domestic portfolio materialized} \\
&\quad \text{in domestic capital} \\
n_d^* &\equiv \frac{K_d^*}{W} = \text{ share of the domestic portfolio materialized} \\
&\quad \text{in foreign capital,} \\
n_b &\equiv \frac{B}{W} = \text{ share of the domestic portfolio materialized} \\
&\quad \text{in bonds,}
\end{aligned}$$

equation (4.1) can be expressed more conveniently as

$$1 = n_d + n_d^* + n_b, \tag{4.8}$$

and plugging (4.8) into the budget constraint (4.7) we obtain the following dynamic restriction for the resources of the domestic economy

$$\frac{dW}{W} = \psi dt + dw, \tag{4.9}$$

where the deterministic and stochastic parts of the growth rate of assets, dW/W, can be expressed in the following way

$$\psi \equiv (\alpha - \eta)\, n_d + (\alpha^* - \eta)\, n_d^* + \eta - \frac{C}{W} \equiv \rho - \frac{C}{W} \tag{4.10}$$

$$dw \equiv \alpha n_d dy + \alpha^* n_d^* dy^*, \tag{4.11}$$

where $\rho \equiv \alpha n_d + \alpha^* n_d^* + \eta n_b \equiv (\alpha - \eta)\, n_d + (\alpha^* - \eta)\, n_d^* + \eta$ denotes the gross rate of return of the asset portfolio.

The objective of the domestic representative agent is to choose the path of consumption and portfolio shares that maximizes the expected value of the intertemporal utility function (4.6), subject to $W(0) = W_0$, (4.9), (4.10), and (4.11). This optimization is a stochastic optimum control problem.[6] It

[6] To solve problems of stochastic optimum control see, for example, Kamien and Schwartz (1991, section 22), Malliaris and Brock (1982, ch. 2), Obstfeld (1992), or Turnovsky (1997, ch. 9; 2000, ch. 15).

is important to bear in mind that the domestic agent takes as given the rates of return of different assets, as well as the corresponding variances and covariances. However, these parameters will endogenously be determined in the macroeconomic equilibrium we shall obtain. We look for values of the endogenous variables that are not stochastic in equilibrium and then we show that the results validate the initial assumption that equilibrium values are not stochastic.

We introduce a value function, $V(W)$, which is defined as

$$V(W) = Max_{\{C,n_d,n_d^*\}} \ E_0 \int_0^{\infty} \frac{1}{\gamma} C^{\gamma} e^{-\beta t} dt, \tag{4.12}$$

subject to the restrictions (4.9), (4.10), and (4.11) and given initial wealth. The value function in period 0 is the expected value of the discounted sum of instantaneous utilities, evaluated along the optimal path, starting in period 0 in the state $W(0) = W_0$.

Starting from equation (4.12) the value function must satisfy the following equation, known as the Hamilton-Jacobi-Bellman equation of stochastic control theory or, for short, the Bellman equation

$$\beta V(W) = Max_{\{C,n_d,n_d^*\}} \quad \left[\frac{1}{\gamma} C^{\gamma} + V'(W) W \psi + 0.5 V''(W) W^2 \sigma_w^2 \right]. \tag{4.13}$$

The right hand side of equation (4.13) is partially differentiated with respect to C, n_d and n_d^* in order to get the first order optimality conditions of this problem

$$\begin{aligned} C^{\gamma-1} - V'(W) &= 0 & (4.14) \\ V'(W) W (\alpha - \eta) dt + V''(W) W^2 cov(dw, \alpha dy) &= 0 & (4.15) \\ V'(W) W (\alpha^* - \eta) dt + V''(W) W^2 cov(dw, \alpha^* dy^*) &= 0. & (4.16) \end{aligned}$$

The solution to this problem is obtained through trial and error. We seek to find a value function $V(W)$ that satisfies, on the one hand, the first order optimality conditions and, on the other, the Bellman equation. In the case of isoelastic utility functions the value function has the same form of the utility function [Merton (1969), then generalized in Merton (1971)]. Thus, we guess that the value function is of the form

$$V(W) = AW^{\gamma}, \tag{4.17}$$

where the coefficient A has to be determined. This guess implies that

$$\begin{aligned} V'(W) &= A\gamma W^{\gamma-1} \\ V''(W) &= A\gamma(\gamma-1)W^{\gamma-2}. \end{aligned}$$

Substituting these expressions in the first order optimality conditions (4.14), (4.15) and (4.16) we get that

$$\begin{aligned} C^{\gamma-1} &= A\gamma W^{\gamma-1} && (4.18) \\ (\alpha-\eta)\,dt &= (1-\gamma)\,cov\,(dw, \alpha dy) && (4.19) \\ (\alpha^{*}-\eta)\,dt &= (1-\gamma)\,cov\,(dw, \alpha^{*}dy^{*}). && (4.20) \end{aligned}$$

These are typical equations in stochastic models in continuous time. Equation (4.18) indicates that at the optimum, the marginal utility derived from consumption must be equal to the marginal change in the value function or the marginal utility of wealth. Equations (4.19) and (4.20) show that the optimal choice of portfolio shares of the domestic representative agent must be such that the risk-adjusted rates of return of assets are equalized.

Combining (4.18), (4.19), and (4.20) and inserting them in the equation (4.13), we can calculate, after some algebra, the equilibrium portfolio shares (implicitly) and the consumption-wealth ratio in the domestic open economy

$$\begin{aligned} \alpha-\eta &= (1-\gamma)\left[n_d\alpha^2\sigma_y^2 + n_d^{*}\alpha\alpha^{*}\sigma_{yy^{*}}\right] \\ \alpha^{*}-\eta &= (1-\gamma)\left[n_d^{*}\alpha^{*^2}\sigma_{y^{*}}^2 + n_d\alpha\alpha^{*}\sigma_{yy^{*}}\right] \\ n_b &= 1-n_d-n_d^{*} \\ \frac{C}{W} &= \frac{1}{1-\gamma}\left[\beta-\gamma\rho+0.5\gamma(1-\gamma)\sigma_w^2\right], && (4.21) \end{aligned}$$

where

$$\sigma_w^2 = n_d^2\alpha^2\sigma_y^2 + 2n_d n_d^{*}\alpha\alpha^{*}\sigma_{yy^{*}} + n_d^{*^2}\alpha^{*^2}\sigma_{y^{*}}^2.$$

To guarantee that consumption is positive in the domestic open economy we impose the feasibility condition that the marginal propensity to consume out of wealth must be positive since wealth does not become negative

$$\frac{1}{1-\gamma}\left[\beta-\gamma\rho+0.5\gamma\left(1-\gamma\right)\sigma_w^2\right]>0.$$

For the first order optimality conditions to characterize a maximum, the corresponding second order condition must be satisfied, that is, the Hessian matrix associated to the maximization problem and evaluated at the optimal values of the choice variables

$$\begin{bmatrix} \left(\gamma-1\right)\left(V'(W)\right)^{\frac{\gamma-2}{\gamma-1}} & 0 \\ 0 & V''(W)W^2\Delta \end{bmatrix}$$

must be negative definite,[7] which implies that

$$\begin{aligned} \left(\gamma-1\right)\left(V'(W)\right)^{\frac{\gamma-2}{\gamma-1}} &< 0 \\ V''(W)W^2\Delta &< 0, \end{aligned}$$

where $\Delta=\alpha^2\sigma_y^2+2\alpha\alpha^*\sigma_{yy^*}+{\alpha^*}^2\sigma_{y^*}^2>0$. To evaluate those conditions first we obtain the value of the coefficient A in equation (4.18)

$$A=\frac{1}{\gamma}\left(\frac{C}{W}\right)^{\gamma-1}, \tag{4.22}$$

where C/W is the optimal value pointed out by equation (4.21). Then substituting expression (4.22) into the value function (4.17), we get that the value function is given, after some algebra, by

$$V(W)=\frac{1}{\gamma}\left(\frac{C}{W}\right)^{\gamma-1}W^{\gamma}, \tag{4.23}$$

where we can observe that, given the restrictions on the utility function, $V'(W)>0$ and $V''(W)<0$ provided that $C/W>0$.

[7] See Chiang (1984, pp. 320-323), for example.

In addition, we impose that the macroeconomic equilibrium must satisfy the transversality condition so as to guarantee the convergence of the value function

$$\lim_{t\to\infty} E\left[V\left(W\right)e^{-\beta t}\right] = 0. \tag{4.24}$$

Now let us show that should the feasibility condition be satisfied then that would be equivalent to satisfy the transversality condition.[8] To evaluate (4.24), we start expressing the dynamics of the accumulation of wealth

$$dW = \psi W dt + W dw. \tag{4.25}$$

The solution to equation (4.25), starting from the initial wealth $W(0)$, is[9]

$$W(t) = W(0)e^{(\psi-0.5\sigma_w^2)t+w(t)-w(0)}.$$

Since the increments of w are temporally independent and are normally distributed then[10]

$$\begin{aligned} E[AW^{\gamma}e^{-\beta t}] &= E[AW(0)^{\gamma}e^{\gamma(\psi-0.5\sigma_w^2)t+\gamma[w(t)-w(0)]-\beta t}] \\ &= AW(0)^{\gamma(1+\eta)}e^{\left[\gamma(\psi-0.5\sigma_w^2)+0.5\gamma^2\sigma_w^2-\beta\right]t}. \end{aligned}$$

The transversality condition (4.24) will be satisfied if and only if

$$\gamma\left[\psi - 0.5\gamma\left(1-\gamma\right)\sigma_w^2\right] - \beta < 0.$$

Now substituting equations (4.10) and (4.21), it can be shown that this condition is equivalent to

$$\frac{C}{W} > 0,$$

and thus feasibility guarantees convergence as well.

[8] See Merton (1969). Turnovsky (2000) provides, for example, the proof of the transversality condition as well.

[9] See Malliaris and Brock (1982, pp. 135-136), for example.

[10] See Malliaris and Brock (1982, pp. 137-138), for example.

In the equilibrium achieved all the assets in the domestic economy grow at the same rate, in addition to the fact that consumption-wealth ratio and portfolio shares are constant[11]. That is the point of departure from the traditional rule. In fact, in contrast to the traditional rule, KV assume that "the country invests the marginal unit of wealth as the average one" (p. 1138), which they term "the new rule". Thus

$$\frac{dK_d}{dW} = \frac{K_d}{W} \tag{4.26}$$
$$\frac{dK_d^*}{dW} = \frac{K_d^*}{W} \tag{4.27}$$
$$\frac{dB}{dW} = \frac{B}{W}, \tag{4.28}$$

which is equivalent to saying that all the assets grow at the same rate

$$\frac{dW}{W} = \frac{dK_d}{K_d} = \frac{dK_d^*}{K_d^*} = \frac{dB}{B}.$$

Analogously, if we assume that all the real assets on the foreign economy grow at another rate,[12] then the current account, via equations (4.1), (4.3), (4.5), (4.26), (4.27), and (4.28), is given by

$$\begin{aligned} CA &= dW\left(\frac{K_d^* + B}{W}\right) - dW^*\frac{K_f}{W^*} \\ &= dW\frac{P}{W} + K_f\left(\frac{dW}{W} - \frac{dW^*}{W^*}\right), \end{aligned} \tag{4.29}$$

where equation (4.29) is equal to that shown in Turnovsky (1997, p. 436), except for the existence of risk-free loans here. Thus the variation in the

[11] With more general utility functions, portfolio shares and consumption-wealth ratio depend on time. However, in our model they are constants because the utility function has constant relative risk aversion, the production function is linear, and the mean and variances of the underlying stochastic processes are stationary. See Turnovsky (1997, p. 370, footnote 6) for more details.

[12] A formal difficulty arises here. The point is that if B grows at the rate of the domestic economy in equilibrium, then the portfolio shares of the foreign economy cannot be constant in equilibrium unless both economies grow at the same rate. If we focus on capital only the difficulty disappears and we obtain basically the same result.

current account when a transitory income shock occurs is equal to the change in savings generated by the income shock minus the investment in domestic capital made by domestic and foreign agents. Please note that the change in domestic wealth is equal to national savings, $dW = S$, as shown in section 4.2.1 above. Under the traditional rule, the investment made by domestic and foreign agents was zero, that is, the domestic agent invests all in foreign assets marginally and it is assumed that the holdings of domestic capital by the foreign agent are constant. We can observe in equation (4.29) that the impact of a transitory income shock on the current account is equal to the amount of savings generated by the shock multiplied by the net foreign asset position of the domestic economy over domestic wealth, on the one hand, plus the difference in the rates of growth of wealth between domestic and foreign economies multiplied by foreign holdings of domestic capital, on the other hand.

First, if the rates of growth of assets of both countries are equal or do not differ very much, then we can see in equation (4.29) that

$$CA = dW\left(\frac{P}{W}\right),$$

where "the current account response is equal to the saving generated by the shock multiplied by the country's share of foreign assets in total assets" (KV, p. 1137). Therefore, the new rule suggested by KV implies a response which depends on the net foreign asset position of the country, that is, whether the country is creditor or debtor: a positive income shock in a creditor nation improves the current account of the country but less than the traditional rule, whereas in the case of a debtor nation a positive income shock deteriorates the current account, in contrast to the result shown by the traditional rule.

Second, if the growth rates of wealth of the domestic and foreign economies are different, then the impact of transitory income shocks on the current account is different from the new rule: the impact of transitory income shocks on the current account depends, in addition to the new rule, on the holdings of domestic capital by the foreign economy and on the difference between the growth rates of wealth in domestic and foreign economies. Thus, the impact of transitory income shocks on the current account follows a more general pattern than that shown in KV. We have termed the new pattern "the extended new rule". If both growth rates of wealth are equal then the extended new rule becomes the new rule, that is, the new rule is a particular case of the extended new rule. However, if growth rates are not equal, then we could expect that the impact would be different from the new rule. Now we can test whether the empirical evidence confirms the traditional rule, the new rule or the extended new rule.

4.3 Data sources

We use the same data set on which KV constructed their paper and thus we refer to their Appendix 2 to get rigorous information on data sources. In this respect they consider that "although data on current accounts and saving are available for many more countries and years, we restrict the sample to those countries for which data on stocks of foreign assets are also available, in order to ensure that our tests of the traditional rule and the new rule are comparable" (KV, p. 1151, footnote 11). The data is based on an unbalanced sample of 13 OECD countries for the period 1973-1995: Australia, Austria, Canada, Germany (1975-1989), Spain (1975-1995), Finland (1975-1995), France (1989-1995), Italy, Japan (1979-1995), Netherlands (1982-1995), Sweden (1982-1995), the United Kingdom, and the United States. We will test the extended new rule with the same sample. However, testing the extended new rule implies some problems. It requires having information on the growth rate of assets of the foreign economy (that is, from the rest of the world) and thus on her level of wealth and amount of savings. That literally is not possible, but we reinterpret the rest of the world as the rest of the sample. Being the sample in KV unbalanced, including or not some countries in the sample can produce significant changes (positive or negative) in some variables (the amount of wealth and savings will differ significantly from year to year, simply due to a change in the number of countries available for that year) and, therefore, testing the extended new rule can be subject to more variability. That is why, in addition to testing based on their sample, we think it is reasonable to test using a balanced sample as well. We show the results for the balanced sample in Appendix A. We should note that the magnitudes of wealth and savings for the rest of the sample are obviously much smaller than those for the rest of the world, S^*/W^*. However, the savings-wealth ratio for the rest of the sample would be a good proxy for that magnitude for the rest of the world if there exists a more or less constant proportionality between the magnitudes of the rest of the sample and those of the rest of the world.

Table 4.1: Net foreign asset position (over domestic wealth)

	Measure one P_{KV} (a)	Measure two P (b)	Difference (a)-(b)
Australia	-0.031	-0.126	0.095
Austria	+0.003	-0.019	0.022
Canada	-0.046	–0.138	0.092
Germany	+0.049	+0.028	0.021
Spain	-0.016	-0.067	0.051
Finland	-0.070	-0.086	0.016
France	+0.059	-0.009	0.068
Italy	+0.009	-0.007	0.016
Japan	+0.044	+0.031	0.013
Netherlands	+0.210	+0.070	0.140
Sweden	-0.050	-0.096	0.046
United Kingdom	+0.123	+0.034	0.089
USA	+0.041	+0.005	0.036
No. of creditor countries	8	5	-3
No. of debtor countries	5	8	+3

Two additional important details apply here. First, the net foreign asset position in this paper, P, is defined by equation (4.3): it is equal to the domestic claims on foreign capital minus foreign claims on domestic capital plus the net position on loans. In contrast, the country's share of foreign assets in total assets in KV, P_{KV}, refers to the domestic claims on foreign capital plus the net position on loans, that is, they do not include foreign claims on domestic capital

$$P_{KV} \equiv K_d^* + B \equiv P + K_f. \tag{4.30}$$

The difference is the direct result of the small open economy assumption in KV. Table 4.1 shows the net foreign asset position of the thirteen countries, with respect to the level of the domestic wealth of the country, based on both measures of net foreign asset position, P_{KV} and P, respectively. We can observe that there is a substantial difference between both types of measures. This difference will be relevant when we test the rules empirically, as we will show below.

Second, we should note that the growth rate of assets of the domestic economy, S/W, and the growth rate of assets of the foreign economy, S^*/W^*, do tend to be different. We show the key properties of the variables S/W and S^*/W^* in Table 4.2: we find that we can reject that both variables have the same mean values for Austria, Spain, Japan, Sweden, United Kingdom, and United States. Some comments need to be done. First, we are aware that the test for mean equality applies for random samples, which is not the case here. Additionally, we should observe that, even in those cases where the null hypothesis is not rejected, both series clearly have very different characteristics for most countries and show significant differences comparing contemporaneous values. We can look at the temporal evolution of the variables S/W and S^*/W^* for the unbalanced sample in Appendix B.

4.4 Empirical evidence

Now, the results of the traditional rule and the new rule in KV are shown, and then the extended new rule tested.

4.4.1 The traditional rule

Following KV we test the traditional rule following via the following regression equation

$$CA_{ct} = \alpha + \beta S_{ct} + u_{ct}, \tag{4.31}$$

where S_{ct} denotes the amount of savings for country c in period t, and u_{ct} is the error term for country c in period t. Under the null hypothesis that the traditional rule is true then the parameter β should be equal to one: an increase in savings leads to a one-to-one increase in the current account. We should point out two differences between this test and the usual followed in the literature. First, Feldstein and Horioka (1980)[13] regressed investment

[13] Feldstein and Horioka (1980, p. 317) wanted to "[...] measure the extent to which a higher domestic saving rate in a country is associated with a higher rate of domestic investment.", so that "with perfect world capital mobility, there should be no relation between domestic saving and domestic investment: saving in each country responds to the worldwide opportunities for investment while investment in that country is financed by the worldwide pool of capital." They find that the empirical evidence runs in favour of a strong

Table 4.2: Key properties of the series S/W and S*/W*

	Mean value of S/W	Mean value of S^*/W^*	p-value for null hypothesis that both means are equal
Australia	0.074	0.074	0.979
	(0.022)	(0.008)	
Austria	0.088	0.074	0.005
	(0.022)	(0.008)	
Canada	0.073	0.074	0.773
	(0.017)	(0.008)	
Germany	0.076	0.072	0.420
	(0.017)	(0.008)	
Spain	0.089	0.074	0.005
	(0.022)	(0.007)	
Finland	0.074	0.074	0.950
	(0.019)	(0.008)	
France	0.075	0.079	0.264
	(0.007)	(0.002)	
Italy	0.079	0.074	0.144
	(0.014)	(0.007)	
Japan	0.110	0.064	0.000
	(0.017)	(0.007)	
Netherlands	0.074	0.073	0.724
	(0.014)	(0.008)	
Sweden	0.060	0.073	0.013
	(0.017)	(0.008)	
UK	0.067	0.075	0.009
	(0.010)	(0.008)	
USA	0.060	0.087	0.000
	(0.007)	(0.012)	

Standard errors are in parenthesis.

on savings. Instead, KV regressed the current account on savings, making it easier to compare the new rule with the traditional rule. However, both approaches are equivalent. Second, Feldstein and Horioka (1980) used data related to Gross Domestic Product, whereas KV have used data related to Gross National Product. We follow the approach used by KV in order to carry out similar comparisons.

We show in Table 4.3 the results of fitting equation (4.31) by OLS, shown in KV. We reject the null hypothesis that the coefficient β is equal to one, that is, we reject the traditional rule. That is, of course, another evidence in favor of the Feldstein-Horioka puzzle. Additionally, we show the between-group estimates (that is, based on the mean values of the variables of the group) and the within-group estimates (also called fixed-effects estimators, that is, in terms of deviations from the mean values of the variables of the group). In any case the null hypothesis that the traditional rule is true is rejected.

4.4.2 The new rule

We test the new rule following KV again

$$CA_{ct} = \alpha + \beta \frac{P_{ct}}{W_{ct}} S_{ct} + u_{ct}. \tag{4.32}$$

---

relationship between both variables, thus attributing it to the lack of perfect world capital mobility. According to Frankel (1992, p. 41), “Feldstein and Horioka upset conventional wisdom in 1980 when they concluded that changes in countries’ rate of national saving had very large effects on their rates of investment and interpreted this finding as evidence of low capital mobility”. However, many economists do not share Feldstein and Horioka’s conclusion. The paradox of having perfect capital mobility going along with a strong association between savings and investment has been termed the “Feldstein-Horioka puzzle”. Many studies followed suit and analyzed the reasons to explain the evidence, while assuming perfect world capital mobility. However, “it seems likely that of many potential explanations of the Feldstein-Horioka results, no single one fully explains the behavior of all countries”, according to Obstfeld and Rogoff (1995, p. 1779). We should note that Feldstein and Horioka (1980, p. 319) were aware that a high association “could reflect other common causes of the variation in both saving and investment”, but they argue that a high association “would however be strong evidence against the hypothesis of perfect capital mobility and would place on the defenders of that hypothesis the burden of identifying such common causal factors.” Finally, recent empirical studies suggest that the Feldstein-Horioka finding seems to be losing some support in the euro area; see Blanchard and Giavazzi (2002).

Table 4.3: The traditional rule

Pooled regression	
Gross national saving/GNP	0.236
	(0.061)
R^2	0.158
Number of observations	247
p-value for $\beta = 1$	0.000
Between-group regression	
Gross national saving/GNP	0.265
	(0.073)
R^2	0.251
Number of observations	13
p-value for $\beta = 1$	0.000
Within-group regression/Fixed effects	
Gross national saving/GNP	0.193
	(0.049)
R^2	0.569
Number of observations	247
p-value for $\beta = 1$	0.000

Standard errors are in parenthesis.

Under the null hypothesis that the new rule is true then the parameter β should be equal to one: increases in savings lead to variations in the current account that are equal to the fraction of the net foreign asset position for country c in period t with respect to the level of domestic wealth for country c in period t.

First, we show in Table 4.4 the results of fitting equation (4.32) by OLS for the sample of thirteen countries, using both the net foreign asset position defined by KV,[14] P_{KV} (first definition, from here onwards), and the net foreign asset position defined by us, P (second definition, from here onwards). We observe that the null hypothesis that the coefficient β is equal to 1, that is, the new rule, cannot be rejected in any of both cases. In general we can see that the estimation using the first definition is closer to 1 compared with the result obtained using the second definition. That is confirmed by a higher p-value as well. However, the goodness-of-fit is slightly better using the second definition than the first. Similar conclusions apply for the between-group and within-group estimations. Additionally, using the first (second) definition most of the variation of the variables is within-(between-)group.

4.4.3 The extended new rule

Following the discussion in section 4.2 above we test the extended new rule, given by equation (4.29), making use of the regression equation

$$CA_{ct} = \alpha + \beta \frac{P_{ct}}{W_{ct}} S_{ct} + \gamma \frac{K_{f,ct}}{W^*_{ct}} S^*_{ct} + \delta \frac{K_{f,ct}}{W_{ct}} S_{ct} + u_{ct}. \qquad (4.33)$$

Under the null hypothesis that the extended new rule is true then β should be equal to one, γ should be equal to minus one, and δ should be equal to one. If the null hypothesis that the new rule were true, then the parameter β should be equal to one, and γ and δ should be equal to zero.

We estimate equation (4.33) by OLS. In Table 4.5 we find that the estimates of the coefficients have the expected signs, and that we cannot reject that $\beta = 1$, $\gamma = -1$, or $\delta = 1$ individually. However, the point estimates fall far from the expected magnitudes and the joint hypothesis that $\beta = 1$, $\gamma = -1$, and $\delta = 1$ is rejected. We should note that the standard errors of the estimates γ and δ are quite high indeed. In addition, we should observe that

[14] Remind the discussion on the net foreign asset position in Section 4.3.

Table 4.4: The new rule

Pooled regression	P_{KV}	P
Gross national saving/GNP		
×Net foreign assets over wealth	0.955	1.164
	(0.078)	(0.105)
R^2	0.369	0.420
Number of observations	247	247
p-value for $\beta = 1$	0.564	0.121
Between-group regression		
Gross national saving/GNP		
×Net foreign assets over wealth	0.996	1.324
	(0.145)	(0.214)
R^2	0.684	0.789
Number of observations	13	13
p-value for $\beta = 1$	0.828	0.158
Within-group regression/Fixed effects		
Gross national saving/GNP		
×Net foreign assets over wealth	0.689	0.655
	(0.187)	(0.284)
R^2	0.563	0.558
Number of observations	247	247
p-value for $\beta = 1$	0.096	0.226

Table 4.5: The extended new rule (I)

	Pooled regression	Between-group regression	Within-group regression
Estimate of β	1.178	1.248	0.831
	(0.097)	(0.160)	(0.276)
Estimate of γ	-1.298	-3.252	-0.647
	(0.357)	(1.625)	(0.416)
Estimate of δ	1.608	3.490	1.025
	(0.336)	(1.594)	(0.393)
R^2	0.467	0.863	0.573
No of observations	247	13	247
p-value for $\beta = 1$	0.068	0.155	0.540
p-value for $\gamma = -1$	0.404	0.199	0.397
p-value for $\delta = 1$	0.072	0.153	0.948
p-value for $\beta = 1$, $\gamma = -1$, $\delta = 1$	0.012	0.212	0.000

the goodness-of-fit increases somewhat with respect to the test of the new rule. If we focus on the between-groups estimation we see very high standard errors, and even though two of the estimates are quite far from the theoretical values, they are not significantly different. The within-groups estimation generates results that resemble those obtained in the pooled regression.

Alternatively, rearranging equation (4.33) under the hypothesis that $\beta = \delta$, the extended new rule can be tested following equation (4.29) as

$$CA_{ct} = \alpha + \beta \frac{P_{KV,ct}}{W_{ct}} S_{ct} + \gamma \frac{K_{f,ct}}{W^*_{ct}} S^*_{ct} + u_{ct}. \tag{4.34}$$

We can observe that under the null hypothesis that the extended new rule is true then β should be equal to one, and γ should be equal to minus one. If the null hypothesis that the new rule (first definition) was true, then β should be equal to one, and γ should be equal to zero. Thus, if $\gamma = 0$, equation (4.34) becomes the new rule (4.32).

Table 4.6 shows the results. The estimates have the expected signs, but we reject the null $\beta = 1$ and the joint hypothesis $\beta = 1$ and $\gamma = -1$. The standard errors of the estimates are much lower than before, and therefore we have improved the precision of the estimates. In addition, the goodness-of-fit

Table 4.6: The extended new rule (II)

	Pooled regression	Between-group regression	Within-group regression
Estimate of β	1.201	1.316	0.880
	(0.094)	(0.173)	(0.222)
Estimate of γ	-0.906	-1.111	-0.492
	(0.143)	(0.304)	(0.267)
R^2	0.464	0.837	0.572
No of observations	247	13	247
p-value for $\beta = 1$	0.033	0.097	0.589
p-value for $\gamma = -1$	0.511	0.723	0.055
p-value for $\beta = 1$, $\gamma = -1$	0.008	0.222	0.000

increases somewhat with respect to the test of the new rule, whereas we get a similar result compared with equation (4.33). The between-group estimation provides a much better fit than before, and none of the hypothesis can be rejected. The within-group estimation generates results that differ considerably from the other estimations. Therefore, the extended new rule adds interesting features, even though the empirical validation is not completely satisfactory.

4.5 Conclusions

The intertemporal approach to the current account is the standard model used today to analyze the impact of real factors on the current account. According to the standard version of the intertemporal approach, or the traditional rule, the impact of a transitory income shock on the current account is equal to the savings generated by the shock $one-to-one$, in all countries, regardless of the net creditor or debtor position of the country. However, the traditional rule fails to account for the empirical evidence on current accounts. KV have proposed recently a remarkably insightful departure from the traditional rule, which they termed the new rule. KV have established that, under the new rule, the impact of a transitory income shock on the current account is equal to the savings generated by the shock multiplied by the net foreign asset position of the country, so that the income shock has a different impact on creditor or debtor economies. In addition, the new rule is a consistent model that brings together the theory and the empirical evidence on current accounts. However, the new rule has been derived from a small open economy model and therefore it ignores some channels through which the foreign economy influences the domestic economy. This paper extends the new rule to a two-country stochastic AK growth model.

First, after reviewing the traditional rule and the new rule, we have shown an extension of the new rule, which we have termed the extended new rule. According to the extended new rule, the impact of a transitory income shock on the current account is equal to the impact suggested by the new rule plus foreign holdings of domestic capital multiplied by the difference between the growth rates of assets in domestic and foreign economies. Thus, only when the domestic and foreign economies grow at the same rate, then the extended new rule becomes the new rule. Therefore, the traditional and the new rule can be understood as particular cases of the extended new rule.

Second, we have tested the traditional rule, the new rule and the extended new rule, based on the unbalanced sample used by KV and a balanced subsample derived from the unbalanced sample. We find that the evidence rejects the traditional rule and thus we have the "Feldstein-Horioka puzzle" again. In order to test the new rule we have used two different measures of net foreign asset position, the one used by KV (first definition) and the another one including foreign claims on domestic capital (second definition). We think that the second definition is more reasonable and compelling. Having tested the new rule using both measures, we found that the results using the first definition are closer to the new rule in the unbalanced sample, whereas the results using the second definition are closer in the balanced sample. However, the goodness-of-fit using the second definition is better in both cases.

Third, we believe that the extended new rule has added important insights to the new rule suggested by KV in order to analyze the impact of transitory income shocks on the current account. Thus the testing based on the data produces estimates that have the expected signs and move around the expected values. That means that the incorporation of the rest of the world cannot be ignored and thus it has to be explicitly modeled. In fact, the rates of growth of the economies do tend to be significantly different. In addition, the goodness of fit of the estimation based on the extended new rule does improve with respect to the new rule. However, it is evident that the data rejects sometimes the null hypothesis that the extended new rule is true. In addition, we should remind that there are important data problems with the estimation of the extended new rule. In a nutshell, we think that the extension of the new rule is a positive step on the road to better understanding the behavior of current accounts. Therefore, even though theoretically we think that the extended new rule provides a good model, the empirical validation of the extended new rule is far from being definitive.

Finally, we would suggest possible avenues for future research. First, the number of countries included in the sample should be extended, since the sample used in this paper has a clear limitation. Second, interesting features have been recently added to the new rule, such as adjustment costs and differences in short run and long run behavior (See Kraay and Ventura, 2002; Ventura, 2003). They could, in turn, be extended to a two-country world. In addition, the extended new rule suggests a possible relation between the rates of growth and creditor/debtor position, which can be empirically tested. That could complement the work by Lane and Milesi-Ferretti (1999, pp. 24-36), where they review the relation between the net foreign asset position and GDP, size and openness to trade, or the "stages hypothesis" suggested by Eichengreen (1991).

Chapter 5

Conclusions and outlook for future research

5.1 Conclusions

Two were the general research objectives of this thesis. The first general research objective was to analyze the role of public spending policy and risk in a two-country world. The second general research objective was to study and empirically test the impact of transitory income shocks on the current account in a two-country world. The common model of analysis is a two-country stochastic AK growth model in continuous time based on Turnovsky (1997, Ch. 11). Now the most relevant results are summarized.

After establishing our general research objectives, justifying the model and reviewing two specific strands of the literature in Chapter 1, we have analyzed the impact of the spending of the public sector and risk on the world economy, provided that public spending is utility-enhancing in Chapter 2. The main results of Chapter 2 (Essay 1) are as follows:

- Most of the results about the impact of changes in exogenous variables (risk, public sector, ...) on the consumption-wealth ratio, the growth rate of wealth, and welfare, provided that the size of the public sector is exogenously given, are standard. However, we should note that a higher weight of public consumption in the utility function raises the growth rate, since consumption-wealth ratio falls. This implies that different preferences towards utility-enhancing government expenditure lead to different rates of growth, *ceteris paribus*. Additionally, increases in the size of the public sector are always growth-reducing, even though they can be welfare-augmenting when the size of the public sector is below its optimal size.

- The results in an open economy have been compared to those of a closed economy, assuming that the size of the public sector is exogenous. Thus, consumption-wealth ratio in an open economy should be higher than that in a closed economy because the lower variance of the growth rate of assets achieved in an open economy encourages consumption. Then we have shown that an open economy will unambiguously grow slower than a closed economy if the marginal physical product of domestic capital is higher than or equal to that of foreign capital. Additionally, since welfare depends upon consumption-wealth ratio, welfare should be higher in a risky open economy than in a risky closed economy, thus extending the results in Obstfeld (1994) and Turnovsky (1997, Ch. 11).

- The welfare-maximizing size of the public sector has been derived. The size of the public sector that maximizes welfare is unambiguously higher than that which maximizes growth. Then we have analyzed the impact of changes in different exogenous variables on the optimal size of the public sector. Whatever increases the variance of the growth rate (a higher covariance between domestic and foreign productivity shocks, for example) reduces the optimal size of the public sector, in contrast to the results found in Turnovsky (1999). In addition, a higher value of the parameter η increases the optimal size of the public sector just in the same amount private consumption-wealth ratio falls, so that public plus private consumption-wealth ratio and the growth rate of wealth do not change. That conclusion differs substantially from the one obtained when the size of the public sector was exogenously given. Finally, the optimal size of the public sector in an open economy should be higher than that in a closed economy under more general conditions than those established in Turnovsky (1999).

Chapter 3 (Essay 2) is devoted to analyze the impact of public spending policy and risk on the world economy, assuming that the spending of the public sector is productivity- and volatility-enhancing. We obtain that:

- The impact of exogenous variables on consumption-wealth ratio, the growth rate of assets and welfare are standard. A higher size of the public sector, enhancing productivity and volatility, changes consumption-wealth ratio. The same ambiguouness can be extended to the growth rate. Welfare depends basically on consumption-wealth ratio, so that public spending influences welfare via consumption-wealth ratio. Even though increasing the size of the public sector increases growth, welfare may fall.

- Comparing the economic results in an open economy with those of a closed economy, we reach the same conclusions we obtained when public spending was utility-enhancing, in Chapter 2 (Essay 1). Thus, consumption-wealth ratio and welfare should be higher in an open economy than in a closed economy and the results on the growth rate depend on the marginal physical product of private capital at home and abroad, and the behavior of consumption-wealth ratio.

- As regards, the optimal size of the public sector in an open economy two different scenarios have been considered because domestic productive government expenditure causes an externality on the foreign economy. In the first scenario the domestic productive public sector is assumed to take into account the impact of productive government spending on the domestic economy only, but not that caused on the foreign economy. In the second scenario the domestic productive public sector is assumed to take into account the impact of productive government spending on both domestic and foreign economies. Then a unilateral and an harmonized optimal size of the public sector in an open economy have been derived, respectively, as well as that corresponding to a domestic closed economy. We have obtained that the optimal size of the public sector in a closed economy with risk is lower than that with no risk, as in Turnovsky (1999). However, in case domestic production risk does not exist, then the harmonized optimal size of the public sector in an open economy does not have to coincide with that in a domestic closed economy, in contrast to Turnovsky (1999).

- The optimal size of the public sector is unambiguously lower than the size of the public sector that maximizes the growth rate due to risk aversion.

- We have compared the optimal size of the public sector in an open economy with that in a closed economy. In the narrower case where the spending of the public sector only influences productivity we reach simpler conclusions. The unilateral optimal size of the domestic open economy should be lower than that in a domestic closed economy because the externality is not completely internalized. In addition, the harmonized optimal size of the public sector in an open economy should be higher than the optimal size in a domestic closed economy if and only if the optimal size of the public sector is higher in a foreign closed economy than in a domestic closed economy: the impact of public spending on productivity is higher in foreign capital than in domestic capital.

- In the more general case public spending influences volatility we have shown that the harmonized optimal size of the public sector in an open economy will be higher than that in a domestic closed economy for two reasons. The first reason has to do with the case public spending is productive-only: the marginal impact of public spending on productivity is higher in a foreign closed economy than in a domestic closed economy. The second reason is due to the higher (that is, less negative) impact of public spending on volatility in an open economy than in a closed economy due to risk diversification. Thus this argument adds new insights to the argument based on the insurance against external risk played by the public sector in Rodrik (1998) and to the risk-exporting argument by Turnovsky (1999).

- Obtaining the harmonized optimal size of the public sector has required that the behavior of both economies is identical, so that the conclusions obtained can be applied to countries where their growth rates are similar.

In Chapter 4 (Essay 3) the impact of transitory income shocks on the current account in a two-country world is analyzed and empirically tested:

- According to the standard intertemporal approach to the current account, or the traditional rule as KV have termed it, the impact of a transitory income shock on the current account is equal to the savings generated by the shock in all countries. That is the result of assuming that the amount saved is totally invested in foreign assets marginally. However, after many years of research, the traditional rule does not seem to be empirically validated.

- KV have suggested an insightful departure from the traditional rule. They have termed it the new rule. They depart from the standard approach postulating that countries invest marginally the amount saved in foreign assets in the same proportion as the average values. Put it another way, this implies all the assets grow at the same rate. Thus, according to the new rule, the impact of a transitory income shock on the current account is equal to the savings generated by the shock multiplied by the proportion of net foreign assets with respect to all domestic assets. The empirical evidence seems to support the new rule.

- The new rule has been extended to a two-country world, so that important channels through which the foreign economy influences on the domestic economy are considered. According to the extended new rule, the impact of a transitory income shock on the current account is equal to the new rule plus the difference between the growth rates of assets in domestic and foreign economies multiplied by holdings of domestic capital owned by the foreign representative agent. Thus, it is the difference between the growth rate of the economies the factor that differences the extended new rule from the new rule. Only when the domestic and foreign wealth grow at the same rate, then the extended new rule becomes the new rule. Therefore, the traditional and the new rule can be understood as particular cases of the extended new rule.

- We have empirically tested the extended new rule, contrasting it to the traditional rule and the new rule, based on an unbalanced sample used by KV for 13 OECD countries in the 1973-1995 period. The growth rates of assets of the countries in the sample tend to be different. We observe that the extended new rule adds important insights to the new rule suggested by KV. However, the extended new rule is not empirically validated in a completely satisfactory way.

5.2 Outlook for future research

The stochastic two-country AK growth model in continuous time developed by Turnovsky (1997, Ch. 11) has provided a deeper understanding about the role of the spending policy of the public sector and risk on the world economy and about the impact of transitory income shocks on the current account. Departing from that model, on the one hand, we have extended the model assuming that public spending is utility-enhancing or productivity-enhancing. On the other hand, we have used it to extend the new rule suggested by KV to a two-country world. However, the model could be extended into other possible interesting directions.

First, most of the work referred to the world economy is based on a two-country framework. Thus, as Turnovsky (1997, p. 209) puts it, “the extension from two to three countries is a major one, and opens up all kinds of interesting questions that are likely to become increasingly relevant in the modern world economy. These relate to issues such as the formation of coalitions of two trading partners and the impact of trading relationships between two countries on a third, and so on”.

Second, focusing on Chapters 2 and 3, the assumption of continuous budget balance impedes to analyze important policy issues such as deficit financing, alternative financing regimes, and so on. Extending the model into that direction would surely add new insights. However, allowing public deficits or introducing money increase enormously the complexity of the model. In addition, since public spending is usually subject to congestion, introducing it would add more realistic features into the productive model in chapter 3.

Finally, as regards Chapter 4, the recent literature on the new rule shows us more realistic features than can be incorporated into the basic model, such as adjustment costs and differences in short run and long run behavior (See KV, 2002; Ventura, 2003). In addition, the empirical validation of the different rules requires a more extensive and complete sample data. However, we should note that the new rule asks for data much harder to find and estimate than the data needed to test the traditional rule. The problem becomes more acute when testing the extended new rule.

Appendix A

Balanced sample results

This appendix shows the results corresponding to the balanced sample. We restrict ourselves to 8 countries and the 1975-1995 period. First, five countries must be dropped out from the sample. The reunification of Germany in 1990 makes non-comparable data before and after the event. Additionally, data for France is only available for 1989-1995, Japan for 1979-1995, Netherlands for 1982-1995, and Sweden for 1982-1995. Second, for the remaining countries we have complete data from 1975-1995. Summing up, we have chosen 8 countries for our sample, namely, Australia, Austria, Canada, Spain, Finland, United Kingdom, Italy, and the United States, and the sample period is 1975-1995.

First, we show the net foreign asset position and the key properties of the savings-wealth ratio of the eight countries in Table A.1 and Table A.2, respectively. They resemble very much those of the unbalanced sample.

Second, testing the traditional rule we find that the Feldstein-Horioka puzzle applies again, more so in the balanced sample than in the unbalanced sample. Table A.3 shows the results. The estimates of β in the balanced sample are generally much lower than those of KV and, additionally, the R-squared falls drastically.

Third, Table A.4 shows the results of fitting the new rule. Again the most visible feature is that the estimates of this sample are significantly lower than those obtained with the unbalanced sample in Table 4.4. That is clearly confirmed by lower p-values than those obtained in the unbalanced sample. While in the unbalanced sample estimation the first definition provided estimates closer to 1, now the second definition generates better results to accept the new rule. In fact, using the first definition we find that the null hypothesis that the coefficient β is equal to 1 can be rejected. As be-

Table A.1: Net foreign asset position (over domestic wealth)

	Measure one P_{KV} (a)	Measure two P (b)	Difference (a)-(b)
Australia	-0.034	-0.127	0.093
Austria	+0.001	-0.021	0.022
Canada	-0.051	–0.138	0.087
Spain	-0.016	-0.067	0.051
Finland	-0.070	-0.086	0.016
United Kingdom	+0.126	+0.034	0.092
Italy	+0.009	-0.007	0.016
USA	+0.041	+0.005	0.036
No. of creditor countries	4	2	+2
No. of debtor countries	4	6	-2

Table A.2: Key properties of the series S/W and S*/W*

	Mean value of S/W	Mean value of S^*/W^*	p-value for null hypothesis that both means are equal
Australia	0.0696	0.0637	0.1055
	(0.0151)	(0.0063)	
Austria	0.0838	0.0636	0.0000
	(0.0162)	(0.0064)	
Canada	0.0697	0.0635	0.0464
	(0.0121)	(0.0064)	
Spain	0.0887	0.0630	0.0000
	(0.0219)	(0.0061)	
Finland	0.0741	0.0638	0.0219
	(0.0189)	(0.0064)	
UK	0.0664	0.0637	0.2862
	(0.0100)	(0.0063)	
Italy	0.0769	0.0625	0.0000
	(0.0126)	(0.0062)	
USA	0.0590	0.0738	0.0000
	(0.0057)	(0.0113)	

Table A.3: The traditional rule

Pooled regression	
Gross national saving/GNP	0.096
	(0.050)
R^2	0.021
Number of observations	168
p-value for $\beta = 1$	0.000
Between-group regression	
Gross national saving/GNP	-0.002
	(0.149)
R^2	0.000
Number of observations	8
p-value for $\beta = 1$	0.000
Within-group regression/Fixed effects	
Gross national saving/GNP	0.180
	(0.058)
R^2	0.331
Number of observations	168
p-value for $\beta = 1$	0.000

Table A.4: The new rule

Pooled regression	P_{KV}	P
Gross national saving/GNP		
$\times$Net foreign assets over wealth	0.654	0.844
	(0.168)	(0.130)
R^2	0.117	0.252
Number of observations	168	168
p-value for $\beta = 1$	0.048	0.2341
Between-group regression		
Gross national saving/GNP		
$\times$Net foreign assets over wealth	0.695	0.879
	(0.248)	(0.210)
R^2	0.391	0.759
Number of observations	8	8
p-value for $\beta = 1$	0.265	0.585
Within-group regression/Fixed effects		
Gross national saving/GNP		
$\times$Net foreign assets over wealth	0.406	0.697
	(0.368)	(0.343)
R^2	0.298	0.324
Number of observations	168	168
p-value for $\beta = 1$	0.109	0.379

fore, the estimation following the second definition provides a much better goodness-of-fit than the first. Similar comments apply to the between-group and within-group estimation. In addition, we have that the goodness-of-fit of the estimation is better following the second definition than the first in all cases.

Finally, we have the results of the extended new rule in Table A.5 and A.6. On the one hand, fitting equation (4.33), we get similar results to the unbalanced sample (Table 4.5), but the goodness-of-fit falls drastically now again. On the other hand, fitting equation (4.34), we get less optimistic results compared with those of the unbalanced sample (Table 4.6). The estimates are further away from the theoretical values compared to the results obtained in the unbalanced sample. In addition, the goodness-of-fit is worse in the balanced sample than in the unbalanced one.

Table A.5: The extended new rule (I)

	Pooled regression	Between-group regression	Within-group regression
Estimate of β	0.780	0.743	0.616
	(0.159)	(0.214)	(0.413)
Estimate of γ	-1.353	-0.248	-0.421
	(0.641)	(1.831)	(0.808)
Estimate of δ	0.852	-0.367	0.212
	(0.649)	(1.535)	(0.738)
R^2	0.280	0.824	0.325
No. of observations	168	8	168
p-value for $\beta = 1$	0.169	0.296	0.353
p-value for $\gamma = -1$	0.583	0.702	0.475
p-value for $\delta = 1$	0.820	0.424	0.287
p-value for $\beta = 1$, $\gamma = -1$, $\delta = 1$	0.137	0.435	0.000

Table A.6: The extended new rule (II)

	Pooled regression	Between-group regression	Within-group regression
Estimate of β	0.778	0.669	0.567
	(0.158)	(0.140)	(0.395)
Estimate of γ	-1.276	-1.439	-0.861
	(0.191)	(0.604)	(0.421)
R^2	0.280	0.796	0.324
No of observations	168	8	168
p-value for $\beta = 1$	0.162	0.065	0.274
p-value for $\gamma = -1$	0.150	0.500	0.741
p-value for $\beta + \delta = 1$, $\gamma = -1$	0.063	0.310	0.000

Appendix B

Figures: Growth rates of assets

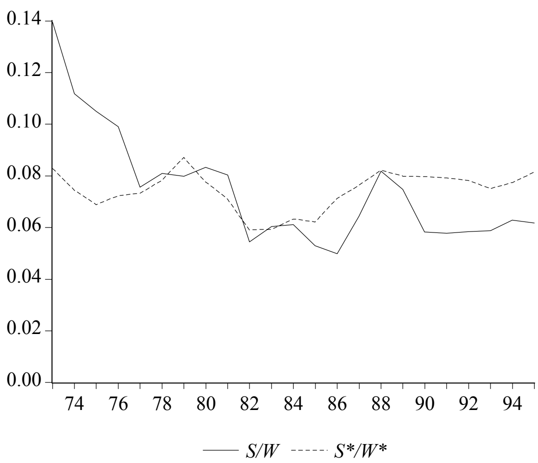

Figure B.1: Growth rate of assets: Australia

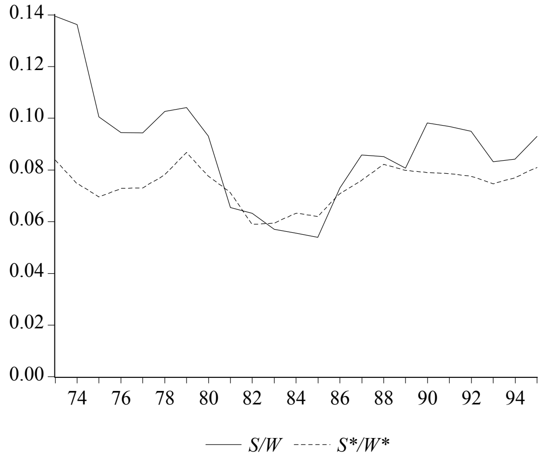

Figure B.2: Growth rate of assets: Austria

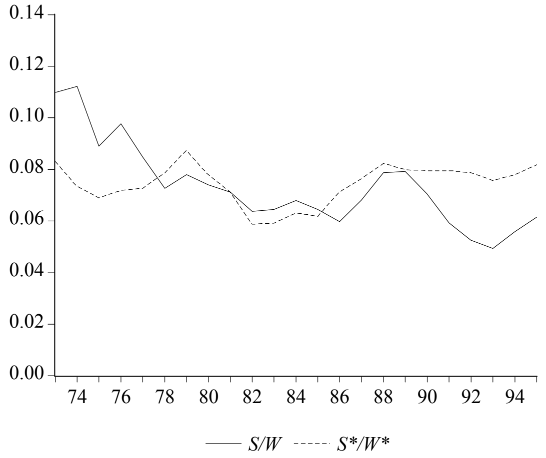

Figure B.3: Growth rate of assets: Canada

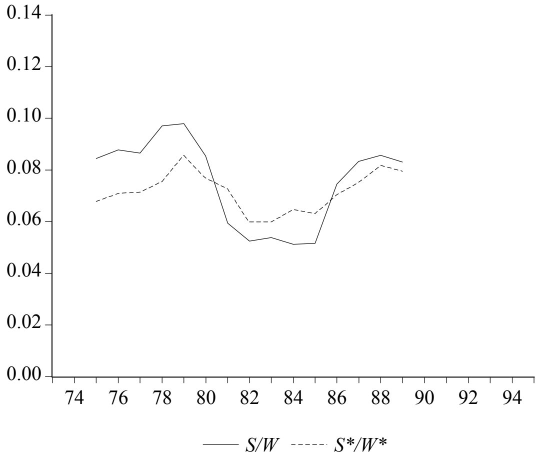

Figure B.4: Growth rate of assets: Germany

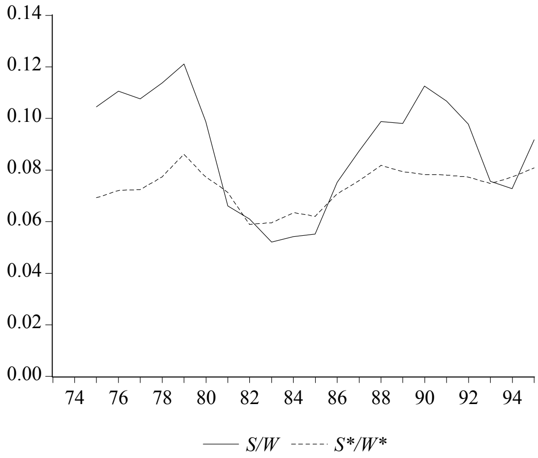

Figure B.5: Growth rate of assets: Spain

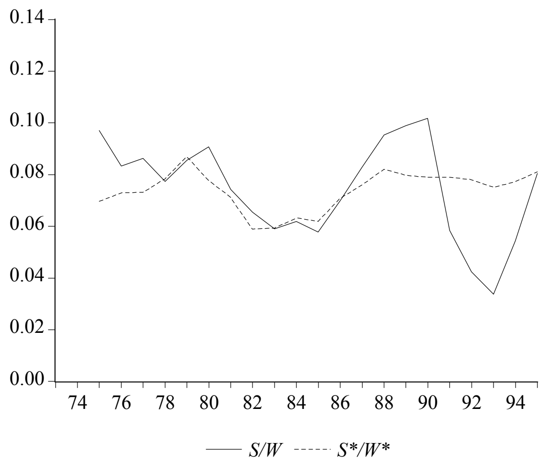

Figure B.6: Growth rate of assets: Finland

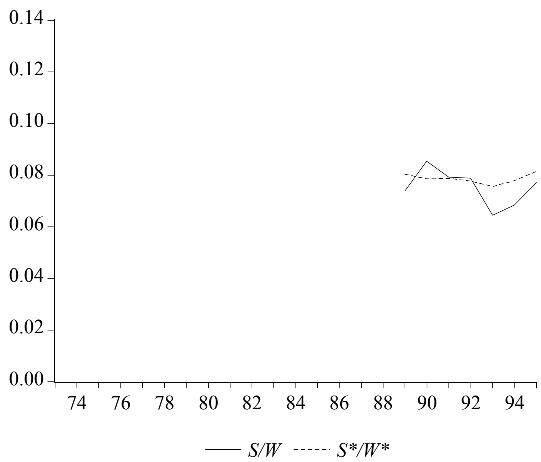

Figure B.7: Growth rate of assets: France

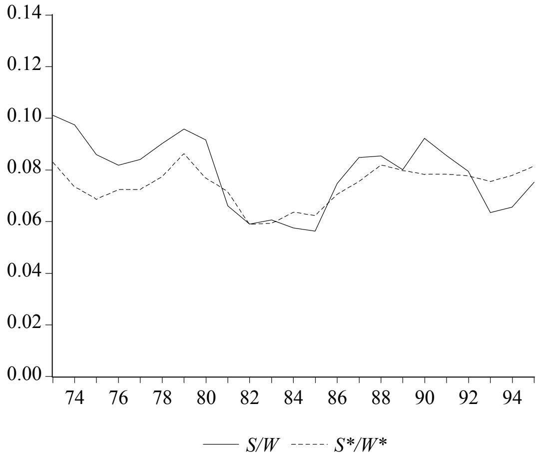

Figure B.8: Growth rate of assets: Italy

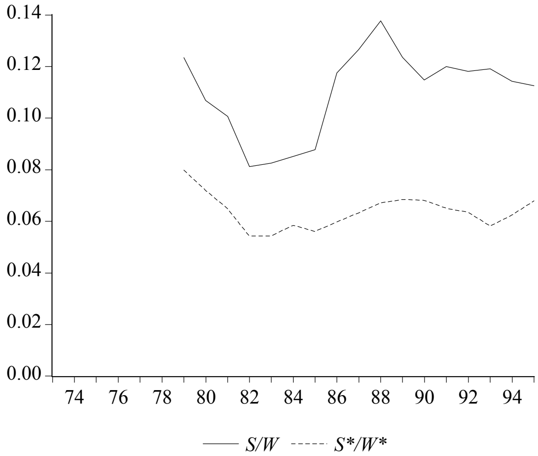

Figure B.9: Growth rate of assets: Japan

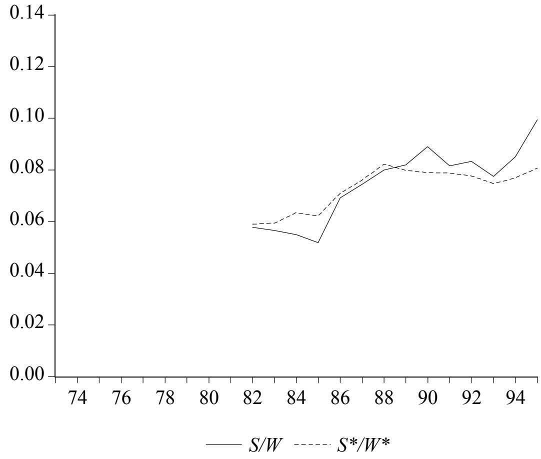

Figure B.10: Growth rate of assets: Netherlands

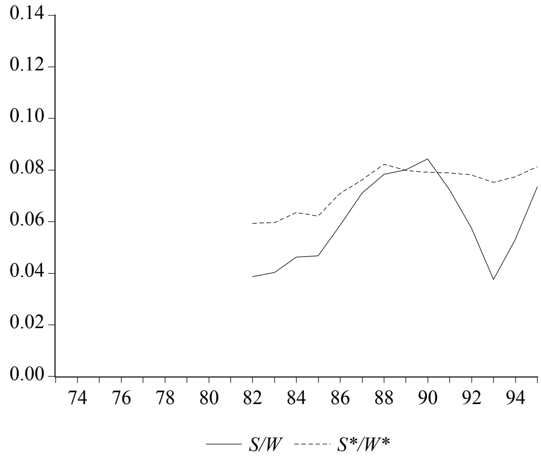

Figure B.11: Growth rate of assets: Sweden

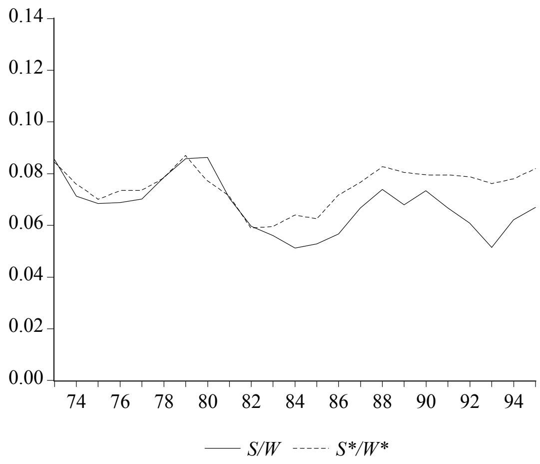

Figure B.12: Growth rate of assets: United Kingdom

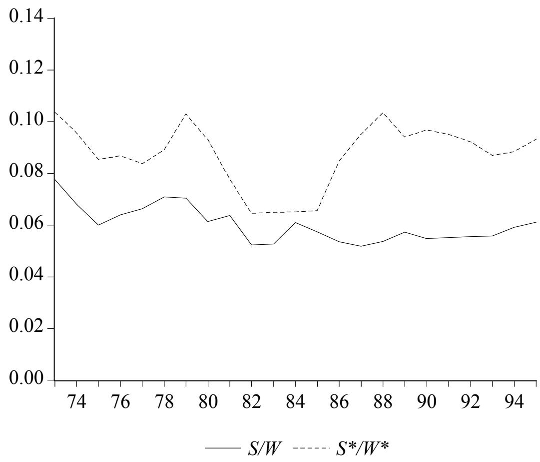

Figure B.13: Growth rate of assets: United States

Bibliography

[1] Aghion, Philippe and Peter Howitt (1998). *Endogenous growth theory.* MIT Press, Cambridge, Massachusetts, United States of America.

[2] Alesina, Alberto and Romain Wacziarg (1998). "Openness, country size and government". *Journal of Public Economics*, 69: 305-321.

[3] Amador, João L. M. (1999). "Fiscal policy and budget deficit stability in a continuous time stochastic economy". Working paper (Universidade Nova de Lisboa, Portugal) available from website *http://www.fe.unl.pt/FE/bibliotecas/BAN/wp-2000.htm.*

[4] Armey, Richard (1995). *The freedom revolution.* Regnery Publishing Company, Washington, D.C., Unites States of America.

[5] Aschauer, David A. (1989). "Is public expenditure productive?". *Journal of Monetary Economics*, 23(2): 177-200.

[6] Baltagi, Badi H. (2001). *Econometric analysis of panel data. Second edition.* John Wiley and Sons, Ltd, Chichester, England.

[7] Barro, Robert J. (1990). "Government spending in a simple model of endogenous growth". *Journal of Political Economy*, 98(5): S103-S125.

[8] Barro, Robert J., and Xavier Sala-i-Martin (1992). "Public finance in models of economic growth", *Review of Economic Studies*, 59(4): 645-661.

[9] Barro, Robert J., and Xavier Sala-i-Martin (1995). *Economic growth.* McGraw-Hill, New York, United States of America.

[10] Barro, Robert J. (1997). *Determinants of economic growth. A cross-country empirical study.* MIT Press, Cambridge, Massachusetts, United States of America.

[11] Blanchard, Olivier J., and Stanley Fischer (1989). *Lectures on macroeconomics.* MIT Press, Cambridge, Massachusetts, United States of America.

[12] Blanchard, Olivier J., and Francesco Giavazzi (2002). "Current account deficits in the euro area: the end of the Feldstein-Horioka puzzle?". *Brookings papers on economic activity*, n^{o}2: 147-209. Brookings Institution, Washington D.C., United States of America.

[13] Burmeister, Edwin, and A. Rodney Dobell (1970) [1973 for the edition in Spanish). *Teorías matemáticas del crecimiento económico.* Antoni Bosch, Barcelona, Spain.

[14] Campbell, John Y. (1996). "Understanding risk and return". *Journal of Political Economy*, 104(2): 298-345.

[15] Chiang, Alpha C. (1984). *Fundamental methods of mathematical economics. Third edition.* McGraw-Hill, Singapore.

[16] Chow, Gregory C. (1997). *Dynamic economics. Optimization by the Lagrange method.* Oxford University Press, New York, United States of America.

[17] Corsetti, Giancarlo (1997). "A portfolio approach to endogenous growth: equilibrium and optimal policy". *Journal of Economic Dynamics and Control*, 21: 1627-1644.

[18] Demmel, Roland (1999). *Fiscal policy, public debt and the debt structure of interest rates.* Springer-Verlag, Berlin Heidelberg, Germany.

[19] Devereaux, Michael B. and Makoto Saito (1997). "Growth and risk-sharing with incomplete international assets markets". *Journal of International Economics*, 42: 453-481.

[20] Dixit, Avinash K. and Robert S. Pyndick (1994). *Investment under uncertainty.* Princeton University Press, Princeton, New Jersey, United States of America.

[21] Domar, Evsey D. (1946). "Capital expansion, rate of growth, and employment". *Econometrica*, 14(2), April: 137-147.

[22] Domar, Evsey D. (1947). "Expansion and employment". *American Economic Review*, 37(1), March: 34-55.

[23] Eaton, Jonathan (1981). "Fiscal policy, inflation, and the accumulation of risky capital". *Review of Economic Studies*, 48: 435-445.

[24] Eichengreen, Barry (1991). "Trends and cycles in foreign lending". In H. Siebert (Ed.), *Capital flows and the world economy.* Tübingen, Mohr, Germany.

[25] Eichengreen, Barry and Charles Wyplosz (1998). "The Stability Pact: more than a minor nuisance?". *Economic Policy*, April: 65-113.

[26] Feldstein, Martin and Charles Horioka (1980). "Domestic savings and international capital flows". *Economic Journal*, 90: 314-329.

[27] Fisher, Irving (1930). *The theory of interest.* Macmillan, New York, United States of America

[28] Fisher, Walter H. and Stephen J. Turnovsky (1998). "Public investment, congestion, and private capital accumulation". *The Economic Journal*, 108, March: 399-413.

[29] Frenkel, Jacob A.; and Assaf Razin, with the collaboration of Chi-Wa Yuen (1996). *Fiscal policies and growth in the world economy. Third edition.* MIT Press, Cambridge, Massachusetts, United States of America.

[30] Fuente, Angel de la (1992). "Histoire d´A: Crecimiento y progreso técnico". *Investigaciones Económicas (2ª época)*, 16(3): 331-391.

[31] Gallaway, Lowell E. and Richard K. Vedder (1995). "The impact of the welfare state on the American economy". Prepared for the Joint Economic Committee of the United States Congress, available from website *http://www.house.gov/jec/.*

[32] Gramlich, Edward M. (1994). "Infrastructure investment: a review essay". *Journal of Economic Literature*, 32:1176-1196.

[33] Greene, William H. (1997). *Econometric analysis. Third edition.* Prentice-Hall, New Jersey, United States of America.

[34] Grinols, Earl A., and Stephen J. Turnovsky (1998). "Risk, optimal government finance and monetary policies in a growing economy". *Economica*, 65: 401-427.

[35] Gwartney, James D., Robert A. Lawson, and Randall G. Holcombe (1998). "The size and functions of government and economic growth". Prepared for the Joint Economic Committee of the United States Congress, available from website *http://www.house.gov/jec/.*

[36] Gylfason, Thorvaldur (1999). *Principles of economic growth.* Oxford University Press, Oxford, United Kingdom.

[37] Hagen, Everett E. (1942). "Capital theory in a system with no agents fixed in quantity". *Journal of Political Economy*, 50: 837-859.

[38] Hall, Robert E. (1978). "Stochastic implications of the life-cycle permanent-income hypothesis: theories and evidence". *Journal of Political Economy*, 86: 971-987.

[39] Harrison, J. Michael (1985). *Brownian motion and stochastic flow systems.* Krieger Publishing Company, Florida, United States of America.

[40] Harrod, Roy F. (1939). "An essay in dynamic theory". *Economic Journal*, 49(193), March: 14-33.

[41] Harrod, Roy F. (1948). *Towards a dynamic economics.* Macmillan, London, United Kingdom.

[42] Hughes Hallet, Andrew; Michael M. Hutchison, and Svend E. Hougaard Jensen (Eds.) (1999). *Fiscal aspects of European monetary integration.* Cambridge University Press, United Kingdom.

[43] Kamien, Morton I., and Nancy L. Schwartz (1991). *Dynamic optimization. Second edition.* North-Holland, United States of America.

[44] Knight, Frank H. (1935). "The ricardian theory of production and distribution". 2 pts. *Canadian Journal of Economics and Political Science*, 1 (February): 3-25; (May): 171-96.

[45] Knight, Frank H. (1944). "Diminishing returns from investment", *Journal of Political Economy*, 52: 26-47.

[46] Kraay, Aart and Jaume Ventura (2000). "Current accounts in debtor and creditor countries", *Quarterly Journal of Economics*, 115:1137-1166.

[47] Kraay, Aart and Jaume Ventura (2002). "Current accounts in the long and short run". *2002 NBER Macroeconomics Annual.*

[48] Kraay, Aart; Norman Loayza; Luis Servén and Jaume Ventura (2000). "Country portfolios". *NBER Working paper series 7795*, available from website *http://www.nber.org/papers/w7795.*

[49] Lane, Philip and Gian Maria Milesi-Ferretti (1999). "The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries". *Working Paper of the International Monetary Fund, WP/99/115.*

[50] Lejour, Arjan M. and Harrie A.A. Verbon (1996). "Fiscal policies and endogenous growth in integrated capital markets". *CentER Discussion Papers* (Tilburg University), available from website *http://center.uvt.nl/pub/dp1996.html.*

[51] Levhari, David and T.N. Srinivasan (1969). "Optimal saving under uncertainty". *Review of economics and statistics*, 36: 153-163.

[52] Lucas, Robert E., Jr. (1988). "On the mechanics of economic development", *Journal of Monetary Economics*, 22, 1 (July): 3-42.

[53] Malliaris, Anastasios G., and William A. Brock (1982). *Stochastic methods in economics and finance.* North-Holland, Amsterdam, The Netherlands.

[54] Malliaris, Anastasios G. (1987). "Stochastic optimal control". In Eatwell, John; Murray Milgate, and Peter Newman (Eds.). *The New Palgrave. A dictionary of economics.* Volume 4. First published in paperback, 1998. Macmillan, London, United Kingdom.

[55] McGrattan, Ellen R. (1998). "A defence of AK growth models". *Federal Reserve Bank of Minneapolis Quarterly Review*, 22(4): 13-27.

[56] Merton, Robert C. (1969). "Lifetime portfolio selection under uncertainty: the continuous-time case", *Review of Economics and Statistics*, 51: 247-257. Reimpressed in Merton, Robert C. (1992). *Continuous-time finance*, Blackwell, Massachusetts, United States of America.

[57] Merton, Robert C. (1971). "Optimum consumption and portfolio rules in a continuous-time model", *Journal of Economic Theory*, 3: 373-413. Reimpressed in Merton, Robert C. (1992). *Continuous-time finance*, Blackwell, Massachusetts, United States of America.

[58] Merton, Robert C. (1987). "Continuous-time stochastic models". In Eatwell, John; Murray Milgate, and Peter Newman (Eds.). *The New Palgrave. A dictionary of economics*. Volume 1. First published in paperback, 1998. Macmillan, London, United Kingdom.

[59] Neumann, John von (1938). "Über ein ökonomisches gleichungssystem und eine verallgemeinerung des brouwerschen fixpunktsatzes", first published in German in the volume entitled *Ergebuisse eines Mathematischen Seminars*, edited by Karl Menger (Vienna, 1938) and translated into English by G. Morgenstern as "A model of general economic equilibrium". *Review of Economic Studies* (1945), 13:1-9. Reproduced in Becker, Robert A. and Edwin Burmeister (Eds.) (1991). *Growth theory.* Edward Elgar, Aldershot, The International Library of Critical writings in Economics, United Kingdom.

[60] Obstfeld, Maurice (1992). "Dynamic optimization in continuous-time economic models (a guide for the perplexed)". Working paper (University of California at Berkeley, United States of America), available from website *http://emlab.berkeley. edu/users/obstfeld/index.html.*

[61] Obstfeld, Maurice (1994). "Risk-taking, global diversification, and growth". *American Economic Review*, 84(5): 1310-1329.

[62] Obstfeld, Maurice and Kenneth Rogoff (1995). "The intertemporal approach to the current account". In Grossman, Gene M., and Kenneth Rogoff (Eds.). *Handbook of international economics. Volume III.* Elsevier Science B.V., Amsterdam, Netherlands.

[63] Obstfeld, Maurice and Kenneth Rogoff (1996). *Foundations of international macroeconomics.* MIT Press, Cambridge, Massachusetts, United States of America.

[64] Obstfeld, Maurice and Kenneth Rogoff (2000). "The six major puzzles in international macroeconomics". *NBER macroeconomics annual, nº15.* MIT Press, Cambridge, Massachusetts, United States of America.

[65] OECD. *National accounts of OECD countries.* Available from website *http://www.sourceoecd.org.*

[66] Razin, Assaf (1995). "The dynamic-optimizing approach to the current account: theory and evidence". In Kenen, Peter B. (Ed.). *Understanding interdependence. The macroeconomics of the open economy.* Princeton University Press, Princeton, New Jersey, United States of America.

[67] Razin, Assaf and Chi-Wa Yuen (1993). "Convergence in growth rates: the role of capital mobility and international taxation", CEPR discussion paper no. 760.

[68] Rebelo, Sergio (1991). "Long-run policy analysis and long-run growth". *Journal of Political Economy*, 99(3): 500-521.

[69] Rodrik, Dani (1998). "Why do more open economies have bigger governments?". *Journal of Political Economy*, 106(5): 997-1032.

[70] Romer, David (2001). *Advanced macroeconomics. Second edition.* McGraw-Hill, New York, United States of America.

[71] Romer, Paul M. (1994). "The origins of endogenous growth", *Journal of Economic Perspectives*, 8(1), Winter: 3-22.

[72] Romer, Paul M. (1986). "Increasing returns and long-run growth", *Journal of Political Economy*, 94(5): 1002-1037.

[73] Rostow, Walt W. (1990). *Theorists of economic growth from David Hume to the present: with a perspective on the next century.* Oxford University Press, New York, United States of America.

[74] Sachs, Jeffrey (1981). "The current account and macroeconomic adjustment in the 1970s". *Brookings papers on economic activity*, I. Brookings Institution. United States of America.

[75] Sala-i-Martin, Xavier (2000). *Apuntes de crecimiento económico. Second edition.* Antoni Bosch, Barcelona, Spain.

[76] Sandmo, Agnar (1970). "The effect of uncertainty on savings decisions", *Review of Economic Studies*, 37: 353-360.

[77] Smith, William T. (1996). "Taxes, uncertainty, and long-term growth". *European Economic Review*, 40: 1647-1664.

[78] Solow, Robert M. (1956). "A contribution to the theory of economic growth". *Quarterly Journal of Economics*, 70(1), February: 65-94.

[79] Solow, Robert M. (1994). "Perspectives on growth theory". *Journal of Economic Perspectives*, 8(1), Winter: 45-54.

[80] Solow, Robert M. (2000). *Growth theory. An exposition.* Second edition. Oxford University Press, New York, United States of America.

[81] Swan, Trevor W. (1956). "Economic growth and capital accumulation". *Economic Record*, 32, November: 334-361.

[82] Turnovsky, Stephen J. (1996). "Fiscal policy, growth, and the macroeconomic performance in a small open economy". *Journal of International Economics*, 40: 41-66.

[83] Turnovsky, Stephen J. (1997). *International macroeconomic dynamics.* MIT Press, Cambridge, Massachusetts, United States of America.

[84] Turnovsky, Stephen J. (1998). "Productive government expenditure in a stochastically growing economy". Available from website *http://www.econ.washington.edu/user/sturn/ /pub_ exp_ and_ stoch_ grow.pdf.*

[85] Turnovsky, Stephen J. (1999). "On the role of government in a stochastically growing open economy". *Journal of Economic Dynamics and Control*, 23: 873-908.

[86] Turnovsky, Stephen J. (2000). *Methods of macroeconomic dynamics.* Second edition. MIT Press, Cambridge, Massachusetts, United States of America.

[87] Turnovsky, Stephen J. (2001). "Growth in an open economy: some recent developments". In Smets, Jan and Michel Dombrecht (Eds.). *How to promote economic growth in the euro area.* Edward Elgar, Cheltenham, United Kingdom.

[88] Turnovsky, Stephen J. (2003). "Old and new growth theories: a unifying structure?". In Salvadori, Neri (Ed.). *Old and new growth theories. An assessment.* Edward Elgar, Cheltenham, United Kingdom.

[89] Valdés, Benigno (1999). *Economic growth. Theory, empirics and policy.* Edward Elgar, Cheltenham, United Kingdom.

[90] Van den Berg, Hendrik (2001). *Economic growth and development. An analysis of our greatest economic achievements and our most exciting challenges.* McGraw-Hill, New York, United States of America.

[91] Vedder, Richard K. and Lowell E. Gallaway (1998). "Government size and economic growth". Prepared for the Joint Economic Committee of the United States Congress, available from website *http://www.house.gov/jec/.*

[92] Ventura, Jaume (2001). "A portfolio view of the U.S. current account deficits", *Brookings Papers on Economic Activity* (July).

[93] Ventura, Jaume (2003). "Towards a theory of current accounts". *The world economy*, 26(4): 483-512.

[94] Wan, Henry Y, Jr. (1971) [1975 for the Spanish edition]. *Teorías modernas del crecimiento. Revisión crítica y resumen.* Vicens-Vives, Barcelona, Spain.

[95] Zagler, Martin and Georg Dürnecker (2003). "Fiscal policy and economic growth". *Journal of Economic Surveys*, 17,3: 397-418.