Research Interests – Enrique Zuazua

1. Research activities

Overall, my field of expertise covers various aspects of Applied Mathematics including Partial Differential Equations (PDE), Numerical Analysis, Control theory and Optimal Design. These interconnected fields have as goal the modelling, analysis, computer simulation and control and design of natural phenomena and engineering processes arising in several contexts of R+D+i.

Some of the contributions of our team are summarized below:

• **Stabilization of systems of vibrations:** Stabilization by means of feedback mechanisms is one of the main topics of Control Theory, given its great mathematical complexity and its important applications in noise attenuation, flexible structures, etc.

- In my Doctoral Thesis, I introduced a new Lyapunov function approach that, by means of multiplier techniques, enables to prove the exponential decay of the solutions to wave equations dissipated on the boundary of the propagation medium. This method caused great interest in the community for its general nature, which was subsequently extended by myself and other groups to different cases, including non-linear wave equations, elasticity systems, Maxwell and Schrödinger equations, etc. As an example of the impact of my work in this field, the following papers are worth mentioning [1], [2].

- This analysis has been extended to the context of thermoelasticity in a series of works where a very rich structure of possible interactions between the thermal and the elastic components has been discovered [3]. We have also developed a quite complete theory for hyperbolic-parabolic problems reminiscent of fluid-structure interaction: [4]. In [5] we have also described the first study on the asymptotic behaviour of fluid-structure interaction systems in the simplified 1-d case.

1 V. Komornik and E. Zuazua (E.Z.), J. Math. Pures et appl. 69(1) (1990), 33-55. (186 citations ISI WoK).

2 E. Z., Comm. in PDE, 15 (2) (1990), 205-235. (190 citations ISI WoK).

3 G. Lebeau and E. Z., Archives Rat. Mech. Anal., 148 (1999), 179-231, (72 cit. ISI WoK).

4 J. Rauch, X. Zhang, E. Z. J. Math. Pures et Appl., (9) 84 (2005), no. 4, 407-470. (28 citations ISI WoK).

5 J. L. Vázquez and E. Z. M3AS, 16 (5) (2006), 637-678. (12 citations ISI WoK).

- Recently we have made a major contribution to the topic of large time asymptotics for hyperbolic systems with partial dissipation, establishing a link with the finite-dimensional control theory. This allowed us in [6] to find a new class of systems in which the decomposition of solutions in decaying components as time tends to infinity varies from the classical one in which the so-called Shizuta-Kawashima condition is fulfilled and solutions may be decomposed into an exponentially decaying component and another one decaying as the heat kernel. Our analysis shows that there are cases in which there is a third term decaying even more slowly, but this only occurs for multi-dimensional problems.

• **Convection-diffusion:** My work concerning the dynamical aspects of PDE brings together several articles on convection-diffusion equations, in which the combination of these two terms produces unexpected non-linear effects. This is a classical subject in Physics, Engineering and Mathematics because of its many applications, and particularly in Fluid Mechanics.

- In [7] we developed a new methodology based on Lyapunov functionals and self-similar variables, enabling the first results on the asymptotic behaviour in various space dimensions to be obtained.

- More recently, in collaboration with J. Vancostenoble, we have adapted these techniques to equations with singular potentials that arise naturally in the linearization of combustion models and in which, from a mathematical perspective, Hardy inequalities play a fundamental role. We have also obtained new results on the possibility of controlling models incorporating these singular potentials both in the context of heat and wave processes. In collaboration with my former PhD student C. Cazacu we also obtained some novel results on Hardy inequalities for multi-polar singular potentials.

- In collaboration with D. Krejcirik we have developed a systematic analysis of the asymptotic behaviour of heat kernel in twisted cylinders.

- The issue of the anisotropic large time asymptotics for hypoelliptic PDEs has been also investigated in a recent paper in collaboration with L. Ignat.

• **Non-linear Control Systems:** One of the great challenges in PDE Control Theory is the move from linear to non-linear systems. This is also a highly relevant problem in applications, since the majority of the most realistic models for natural and technological processes and phenomena are non-linear (elasticity, fluids, structures, materials, etc.). My work has addressed these

6 K. Beauchard & E. Z., Arch. Rational Mech. Anal., 199 (2011) 177–227 (13 citations WoK).
7 M. Escobedo and E. Z., J. Funct. Analysis, 100 (1991), 119-161, (129 citations ISI WoK).

issues in the context of PDE.

- The paper [8] introduces a fixed-point method that is already regarded as classical. By using fine estimates of the cost of controlling linearized problems, it enables control results for wave equations with non-linearities with an optimal growth rate at infinity.

- In [9] this method is adapted to the case of strongly irreversible systems in time, such as heat equations, in which one needs to relax the final control condition to be convert it into an approximate control condition. In paper [10] this work is completed with fine estimations on the cost of control for non-linear models, allowing the possibility of controlling some blowing up mechanisms, in particular.

- More recently we have extended this analysis to viscous Hamilton-Jacobi equations [11]

• **Controllability of linear PDE:** Despite of the advances on the controllability of nonlinear PDE there are still many fundamental questions that are still poorly understood even at the linear level. In the last few years we have got some fundamental results that we summarize below.

- For wave-like equations, in the continuous setting, the observability constants are well known to be closely related to the so-called Geometric Control Condition requiring that all rays of Geometric Optics enter the set of observation on the given time horizon. This problem is not so well understood for heat-like equations. In that context, because of the infinite speed of propagation, observability holds in an arbitrarily small time and without any geometric condition. But because of the time-irreversibility it is well known that an exponential boundary layer arises at the initial time. In [12], using an unexpected and original transformation linking the heat and the wave equation, we prove the first sharp results in that context.

- Due to the strong time-irreversibility of the heat equation the mapping associating the data of the adjoint solution leading to the control to the data to be controlled, does not necessarily preserve the regularity, as it

8 E. Z., Ann. IHP. Analyse non linéaire. 10 (1993), 109-129 (83 cit. ISI WoK).

9 C. Fabre, J. P. Puel and E. Z., Proc. Roy. Soc. Edinburgh, 125A (1995), 31-61, (179 cit. ISI WoK).

10 E. Fernández-Cara and E. Z., Annales de l'IHP, Analyse non linéaire, 17 (5) (2000), 583-616 (109 citations ISI WoK).

11 A. Porretta and E. Z., Annales IHP, Analyse non linéaire, 29 (2012), pp. 301-333.

12 S. Ervedoza and E. Z, ARMA, 202 (2011) 975–1017 (3 citations ISI WoK).

happens for wave like equations, as proved in [13]. In [14], in the case of the heat equation, we prove that the behaviour is the opposite one and, actually, that the data for the adjoint equation are extremely irregular, with exponentially growing Fourier coefficients, even for very smooth data to be controlled.

- Despite the important number of papers dealing with the control of heat like equations, very little is known in the context of hypo-elliptic models. In our paper [15] we make a first contribution to this field, analyzing the controllability of the famous Kolmogorov equation. Our results have attracted much attention and stimulated research of different groups on the control of hypo-elliptic equations.

- In [16] we build examples showing the optimality of the Carleman inequalities often used in the control of heat and wave equations, in what concerns the dependence with respect to lower order potentials.

- More recently we have investigated more complex control patterns, often relevant in applications. In [17] we develop a new theory showing that classical duality tools can also be used to build time-switching controls fulfilling a suitable minimality condition. In a more recent paper in collaboration with Q. Lu we show that, in some instances, the control strategy can be build so to be robust with respect to unknown switching control patterns, a fact that is extremely relevant in applications.

• **Waves in heterogeneous media:** Another subject on which I have worked intensively since 1989, motivated by control theoretical questions, is the propagation of waves in highly heterogeneous media, a field of great importance, particularly in the areas of oil prospecting, acoustics, new materials, etc.

- In paper [18] we developed a fine construction of a fractal nature that enables one to prove that the usual properties of wave propagation and controllability are lost in the framework of Hölder continuous coefficients, thereby demonstrating the optimality of the results for coefficients of

13 S. Ervedoza and E. Zuazua. Discrete Contin. Dyn. Syst. Ser. B, 14(4): 1375--1401, 2010 (6 citations ISI WoK).

14 S. Micu and E. Zuazua, Systems and Control Letters, 60 (2011) 406-413 (2 citations ISI WoK).

15 K. Beauchard and E. Z., Ann. Institut Henri Poincaré, Analyse Non Linéaire, 26 (5) (2009), 1793-1815 (2 citations ISI WoK).

16 Th. Duyckaerts, X Zhang, E. Z., Annales IHP, 25 (2008) 1–41. (33 citations ISI WoK).

17 E. Z., J. Eur. Math. Soc., 2011, 13, 85–117 (8 citations ISI WoK).

18 C. Castro and E. Z., Archive Rational Mechanics and Analysis, 164 (1) (2002), 39-72. (23 cit. ISI WoK).

bounded variation, that we proved previously. This also establishes the limits of the existing dispersive estimates for clusters of eigenfunctions by Smith and Sogge, which play a critical role when solving nonlinear waves and Schrödinger equations in manifolds and bounded domains.

- I have also intensively studied the problem of wave propagation on networks, of primary importance in so many applications as irrigation, traffic, etc. Our contribution in the field is summarized in my Springer 2006 book in collaboration with R. Dáger and my recent survey article to be published in a CIME-Springer volume.

• **Numerics for wave propagation:** My team has developed an intensive research agenda on the interaction between numerical simulation, wave propagation and control, which is of great use in problems concerning noise attenuation, flexible structures, etc. In this area numerical simulations rely on discrete models, which may generate spurious solutions that dramatically modify the propagation and, consequently, control properties.

- In 2005, a review paper published in the prestigious SIAM Review [19] summarized our work since 1998. There, first, we rigorously demonstrate that classical methods may give rise to erroneous numerical approximations in control because of the presence of high spurious frequencies. Secondly, we develop different methods to avoid these spurious numerical solutions by means of dual mesh filtering techniques, mixed finite element methods, numerical viscosity, etc. A series of highly complex analytical problems are also posed. They have motivated much work that is currently being performed by different research teams.

- Later in [20] we have shown that the same analysis applies to other relevant issues, as for instance the efficiency of the method of Perfectly Matched Layers (PML), so relevant in wave computing and simulation. My invited section lecture at International Congress of Mathematicians-ICM06 was devoted to this topic.

- Recently within our research programme in this field we have undertaken the analysis of fully discrete schemes and discontinuous Galerkin methods, among other topics. A Springer-Briefs volume will be published in 2014, in collaboration with A. Marica, with a survey of our contributions in what concerns discontinuous Galerkin methods.

- An updated presentation of our contributions in this field can be found in the 2012 survey paper published in collaboration with S. Ervedoza in the

19 E. Z., SIREV, 47 (2) (2005), 197-243 (102 cites ISI-WoK).

20 S. Ervedoza, E. Z., Numer. Math. 109 (2008), 597—634. (8 citations ISI WoK).

CIME Subseries of Springer, and the Springer Briefs Volume 2013.

- In [21] we have developed a systematic method capable of transferring results on the controllability of time-continuous conservative semigroups into conservative time-discrete versions. This allows getting in a systematic manner convergent fully discrete schemes for the control of waves. In [22] we develop a harmonic analysis method allowing proving the convergence of the two-grid algorithm for the convergence of numerical methods of control of waves.

- In a recent preprint [57], in collaboration with A. Aurora, by means of the analysis of Wigner measures, we develop a microlocal approach to describe the propagation of numerical waves in 1-d non-uniform discrete meshes. We show that high frequency numerical waves propagate according to the bicharacteristic rays associated with a symbol taking account of the heterogeneity of the mesh. In some sense, our results show that the non-uniform grid introduces similar effects as the variables coefficients produce at the continuous level. In particular, bicharacteristics are not straight lines any more, even if the underlying wave equation has constant coefficients. Our numerical experiments confirm fully the results since they exhibit full coincidence between the trajectories of the Hamiltonian system associated to the symbol and the propagation of the support of the high frequency numerical wave-packets.

  This constitutes a very promising research line. Extensions to the multi-dimensional case are ongoing. This work may have also interesting consequences in what concerns mesh-adaptive methods. Indeed, our results constitute a warning about the possible undesired effects that adaptive heterogeneous meshes may have for the propagation of high frequency numerical solutions.

Most of the work in this context has been developed by classical finite-difference and finite-element methods. There are however some earlier interesting contributions in the context of multi-resolution and wavelet methods (see [56]). The results above should definitely be explored in this wavelet context.

• **Numerical flow control.** Motivated by joint work with colleagues at AIRBUS Operations and the Spanish Institute of Aerospace Technology (INTA), during the last 9 years we have worked on shape design problems in aeronautics,

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21 S. Ervedoza, Ch. Zheng, E. Z., J. Functional Analysis, 254 (12) (2008), 3037-3078. (14 citations ISI WoK).

22 L. Ignat, E. Z., J. European Math. Soc., 11 (2009), 351-391 (WoK Citations: 6).

making various contributions that are currently used by AIRBUS in their designs. This work has been developed mainly in collaboration with F. Palacios, now at The Boeing Company. Our research activity in this area has led to publication both in Applied Mathematics Journals, but also to a series of papers published in the AIAA (American Institute of Aeronautics and Astronautics) Journal.

Our main contributions in this area have been as follows:

- We have developed systematically the continuous approach to optimal shape design and developed a novel application to the complete system of compressible 3D Navier-Stokes equations without using domain integrals in the gradient evaluation [23], 2D Euler equations including the derivation of the Rankine-Hugoniot adjoint internal relations [24], and 3D Reynolds-averaged Navier-Stokes equations with the complete derivation of the continuous adjoint method for the Spalart-Allmaras turbulent model [25]. Most of this original work has been implemented in the TAU code used by Airbus Operations for the design of their commercial aircrafts.

- From a theoretical point of view, one of the main concerns is the impact on shape optimization of the shock or shock-like solutions that often arise in the models of fluid mechanics. In[26], we introduced an alternate direction method that distinguishes and combines the effect of the design parameters on the smooth components of solutions and on the location and nature of its singularities, making descent algorithms much more efficient.

- We have extended this analysis in various directions, including viscous problems, steady-state models and multi-dimensional ones.

- We have also shown how the complexity of the boundary functionals to be optimized can be reduced so as to allow numerical methods of lower order to be efficient.

- More recently, in a joint paper with M. Ersoy and E. Feireisl, we have shown that steady state solutions are limits as time tends to infinity of time-evolution ones in the context of one-dimensional scalar conservation laws. We have also developed the corresponding asymptotic analysis for the sensitivity in the presence of shocks. This is a first justification, in a

23 C. Castro, C. Lozano, F. Palacios, and E. Zuazua, AIAA Journal Vol. 45, No. 9, pp. 2125-2139, September 2007 (9 citations WoK).

24 A. Baeza, C. Castro, F. Palacios, and E. Zuazua, AIAA J., 47 (3), 2009 (3 citations WoK).

25 A. Bueno-Orovio, C. Castro, F. Palacios, and E. Zuazua, AIAA J., 50 (3), 2012. (2 citations WoK).

26 C. Castro, F. Palacios and E. Zuazua, M3AS, 18 (3) (2008), 369-416 (11 citations WoK).

simplified setting, of a commonly used reduction in aeronautics when dealing with optimal design problems.

- In a joint paper in collaboration with L. Ignat and A. Pozo[27] we analyse the asymptotic behaviour of different conservative numerical schemes for conservation laws. Using scaling and compactness arguments we show that some schemes, as Lax-Friedrichs, for instance, exhibit a diffusive dynamics, while some others, like Engquist-Osher, mimic correctly the hyperbolic one. This significantly affects the asymptotic behaviour of solutions since one obtains, for the more diffusive schemes, viscous waves rather than the well-known hyperbolic N-wave pattern. At present we are analyzing the important consequences of this fact when dealing with optimal control problems, as those arising in the challenging topic of sonic-boom control for the new generation of supersonic aircrafts.[28]

• **Numerics for heat control.** As seen above, there are a number of efficient methods for computing accurate approximations of controls for wave-like equations. The situation is different for the heat equation where the strong time-irreversibility makes the problem extremely ill-posed. In [29] we have developed a discrete version of the so-called transmutation method, linking the wave and the heat equation. This allows using the numerical controls of the wave equation to get accurate approximations of the controls of the heat equation, bypassing the ill-posedness of the problem. Transmutation has been used in the past in order to write the controls of the heat equation as a function of those of the wave equation, but this paper constitutes the first contribution at the numerical level. Our numerical simulations exhibit a better performance than earlier works based on regularization and penalization methods.

This work has been recently complemented with a contribution in collaboration with R. Devore where we address a different aspect of the time irreversibility of the heat equation, in the context of the inverse problem of identifying the initial source of heat. We have developed an optimal time-sampling algorithm for a thermometer, placed in the interior of the 1-d domain where heat propagates, that allows determining in an optimal manner the initial temperature.

• **Numerical optimal design.** The theory of optimal design, and, in particular, shape design, has undergone significant progress in the last two decades at the theoretical level, often derived using fine tools of geometric measure theory, showing the existence and regularity of optimal shapes for various problems in

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27 L. Ignat, A. Pozo y E. Zuazua, Large-time asymptotics, vanishing viscosity and numerics for 1-D scalar conservation laws, Math of Computation, to appear.

28 J. J. Alonso and M. R. Colonno, Annual Review of Fluid Mechanics, 44 (2012), 505-526.

29 A. Münch and E. Z.. J. Inverse Problems, 26(8) 085018 (2010) (10 citations ISI WoK).

elasticity and fluid mechanics. However, much less is known in what concerns the optimal placement of sensors and actuators for control systems modelled by PDE, despite the relevance of these issues in control systems applications. Significant progress has also been made concerning the development of efficient numerical algorithms based on shape differentiation; gradient and level set methods.

Despite of this, there is still nowadays an important gap between theory and simulations. Indeed, there are very few examples in which it has been rigorously established that optimal numerical shapes converge, as the mesh-parameter tends to zero, to the optimal shape of the PDE model.

- We have developed a systematic research program in this field. In our 2006 paper [30] we have formulated the problem of the convergence of discrete finite element optimal shapes towards the continuous ones in the context of geometric optimal design of Laplace's equation with Dirichlet boundary conditions in 2D and developed a general theory allowing proving it.
- This methodology has been recently adapted to the optimal design of conductivity and elasticity coefficients in elliptic problems. In a recent (2011) SIAM J. Multiscale Analysis paper in collaboration with J. Casado-Díaz, C. Castro and M. Luna-Laynez, we prove the convergence and analyze the convergence rates of finite-element optimal shapes towards the continuous ones. This is done from a multi-scale point of view since, indeed, the mesh in which the PDE and the shapes are discretized are not necessarily the same. In 1-d we compute the optimal coarseness of the mesh in which the coefficients are discretized with respect to the one in which the PDE is discretized. We also prove that, even if relaxation does not occur because the original continuous design problem has a classical solution, using the relaxed version yields faster convergence rates.
- This work complements our previous contribution on the numerical analysis of homogenization problems based on Bloch expansion[31]. Our work has complemented and made more precise other approaches in the field, aimed to develop special Galerkin basis in which the approximation rate is, roughly, independent of the oscillations presented by the coefficients.

This works are to be related to earlier research developed by Ch. Schwab and collaborators in [54] and [55], for instance, in which they develop ad-hoc methods

30 D. Chenais & E. Z. JMPA, 85 (2006), 225-249, (6 citations ISI-WoK).

31 R. Orive, E. Z. Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal, 4 (1) (2005) pp. 36-87(5 citations ISI WoK).

for the numerical approximation of PDE with highly oscillatory coefficients. Their potential use in the effective approximation of optimal design problems is to be investigated.

• **Optimal placement of sensors and actuators.** In collaboration with Y. Privat and E. Trélat, we are developing a systematic research agenda for the determination of the optimal location of sensors and actuators in control systems.

In the context of the wave equation, we have rigorously formulated the problem of searching the optimal location of observers and controllers with a given volume fraction. We have shown that different problems have to be distinguished leading to different conclusions and, in particular, whether one is dealing with a specific solution, or subset of solutions, or all possible solutions of finite energy.

Concerning the last problem, by means of a randomization procedure (looking to most solutions rather than all of them), or making measurements in infinite time, the problem has been reduced to a spectral criterion. We have considered this new optimal design problem involving the whole spectrum of the laplacian together with its relaxed version. The results we prove are essentially as follows. First we show that relaxation occurs, in the sense that the optimal density for the relaxed problem is the constant one. Second, that there is no gap between the original and the relaxed problem. And third, we have proved that a spillover phenomenon occurs so that, typically, the best choice of the location for the sensors for the first N eigenfunctions, is the worst one for the next N+1-th one.

This has been done in full detail in the one-dimensional case. In the multi-dimensional case we have also established an important and deep connection with the geometry of eigenfunctions at high frequencies, the dynamics of the billiard of Geometric Optics and its stability properties.

We have also shown that, when dealing with one individual solution, the optimal set, even if it is achieved, it can be of Cantor type. This observation introduces a subtle difference that numerical experiments exhibiting, frequently, highly oscillatory patterns could not detect. Indeed, often those oscillations were systematically associated with the relaxation phenomenon. But our results show that they can also arise in the case where the optimal location is actually achieved on a complex measurable set of Cantor type.

We have also analyzed the same problem for the heat equation. In this case we show that, due to the intrinsic dissipativity of the model, relaxation and spillover phenomena do not occur and that, actually, the optimal sets are determined by a finite number of Fourier components. We also establish sharp bounds on the number of relevant Fourier components in terms of the observation/control time horizon.

We have written and published several articles and preprints on this topic. This is a very rich area that we are pursuing now in various directions. Among others:

- Optimal feedback controllers
- High frequency filters
- Numerical methods to compute optimal shapes.

• **Averaged control.** Recently we have introduced a new notion for averaged control for systems depending on unknown parameters. The goal is to design control mechanisms in a robust manner, so that the average of all the states with respect to the unknown parameter is under control. We have shown that the dual notion is a property of averaged observability.

In the final dimensional case we have fully characterized these two properties by means of a suitable rank condition. In the PDE frame, in a joint work with M. Lazar submitted for publication to the C. R. Acad. Sci. of Paris, we have first considered wave equations depending on parameters. Using microlocal defect measures we have shown that, under suitable geometric conditions, both average observation and control hold.

This is a very promising subject with various directions of possible development, including the case where the coefficients depend on the unknown parameters in some probabilistic way, so to make the notion of control to be robust with respect to uncertain perturbations. This issue has been previously considered in a joint article in collaboration of Q. Lu in the case where a parabolic dynamics is controlled in randomly switching between two distinct control mechanisms.

• **The turnpike property.** Very often, relevant control problems with applications in Earth and Social Sciences are formulated in long time scales.

In a series of joint works in collaboration with A. Porretta and E. Trélat we have analyzed the problem of the possible asymptotic simplification of controls and controlled trajectories in long time control horizons. The so-called turnpike property refers precisely to this issue and guarantees that, when the time-horizon is long enough, during most of the evolution, the controls and controlled dynamics are close to the optimal steady state control and state.

We have actually proved that this turnpike property holds in a rather general context, both in the final dimensional and for linear PDE.

The case of semilinear PDE is under investigation, together with other important applications to, for instance, shape design, where, in engineering applications, often, the problem is addressed in its steady-state version, expecting that it will yield an appropriate approximation to the optimal shapes for the time-evolution problem.

2. Interactions with industry

I had also the opportunity to work and run a numerous research projects in close collaboration with industry. I briefly describe the most significant ones:

- In collaboration with my former PhD student Francisco Palacios (now at The Boeing Company, Long Beach, CA) I developed an intensive research agenda in aeronautic optimal design and numerical analysis with connections with industry and in particular the Airbus Consortium. This research has lead to the development of the new software Stanford University Unstructured – SU2, http://su2.stanford.edu/

- I was the PI of a joint industrial research project with the Arteche Group (http://www.arteche.com/en) to develop computational software for parameter identification of electrical networks from on-site measurements.

- I also contribute to launch the research group in Computational Fluid Dynamics of Baltogar company: http://www.baltogar.com .

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