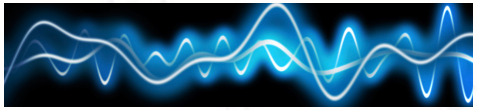
basque center for applied mathematics



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Neure ibilbidea eta lana // My trajectory and work

Enrike Zuazua Iriondo

Ikerbasque Research Professor & Director of the BCAM

Jakiunde, Azaroak 20, 2009

Outline:

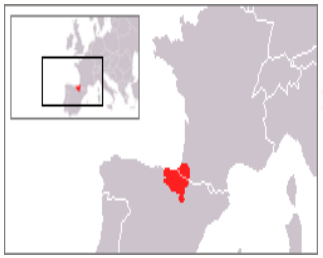
1. My trajectory
2. BCAM
3. My research

1. My trajectory

1. Born in Eibar: September 28, 1961
2. Education: Ikastola - La Salle - Universidad Laboral - UPV-EHU - Université Pierre et Marie Curie - Collège de France
3. PhD: February 1988
4. Assistant Professor UPV-EHU (87-88), Associate Professor UAM (88-90), Professor of Applied Mathematics UCM (1990-2000), UAM (2001-2008), Ikerbasque and BCAM (2008—)
5. Highly Cited Researcher, ISI -Thomson, 2004
6. Euskadi Prize 2006
7. Julio Rey Pastor National Prize (Spain), 2007
8. Jakiunde 2007
9. Awarded the European Research Council Advanced Grant NUMERIWAVES: 2010-2014
10. Cofrade del Bacalao, Eibar, San Andrés 2009

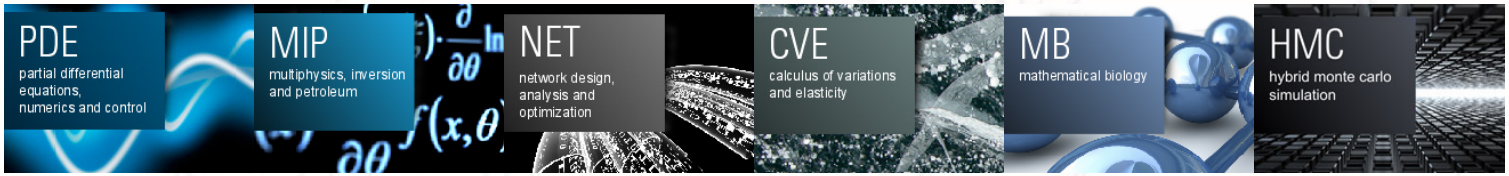
2. BCAM

- The **Basque Center for Applied Matematics, BCAM,** is a research center on applied mathematics promoted by the **Basque Government** through **Ikerbasque**, with the support of **other basque R+D institutions** such as the University of the Basque Country and Innobasque, and other cooperating third Institutions as Bizkaia Xede.
- BCAM is one of the members of the **BERC network: Basque Excellence Research Centers.**
- BCAM aims to strengthen the Basque science and technology system, by performing **research in the frontiers of mathematics, training and attracting talented scientists.**
- BCAM aims to become a **relevant node in the international mathematics research network.**



Research Lines

- PDE – Partial Differential Equations, Numerics and Control Theory
- MIP – Multiphysics, Inversion and Petroleum
- NET – Network Analysis, Design and Optimization
- CVE – Calculus of Variations and Elasticity
- MB – Mathematical Biology
- HMC – Hybrid Monte Carlo Simulations



Scientific Council

- Juan José MANFREDI – U. Pittsburgh (USA) [President]
- Sir John BALL – U. Oxford (UK)
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- Jean-Michel CORON – U. Pierre et Marie Curie & Institut Universitaire de France (France)
- Leszek F. DEMKOWICZ – U. Texas at Austin (USA)
- Pierre-Louis LIONS – Collège de France (France)

Come and visit us!

basque center for applied mathematics

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3. My research

Goals:

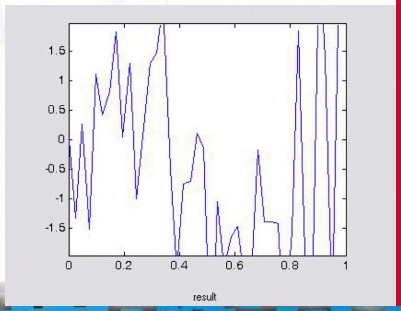
- To develop analytical and numerical methods allowing to mimic and reproduce fine qualitative properties of solutions to Partial Differential Equations (PDE), oriented, in particular, towards design and control applications: aeronautics, networks (irrigation, gas, rivers,...), complex structures,...

Tools:

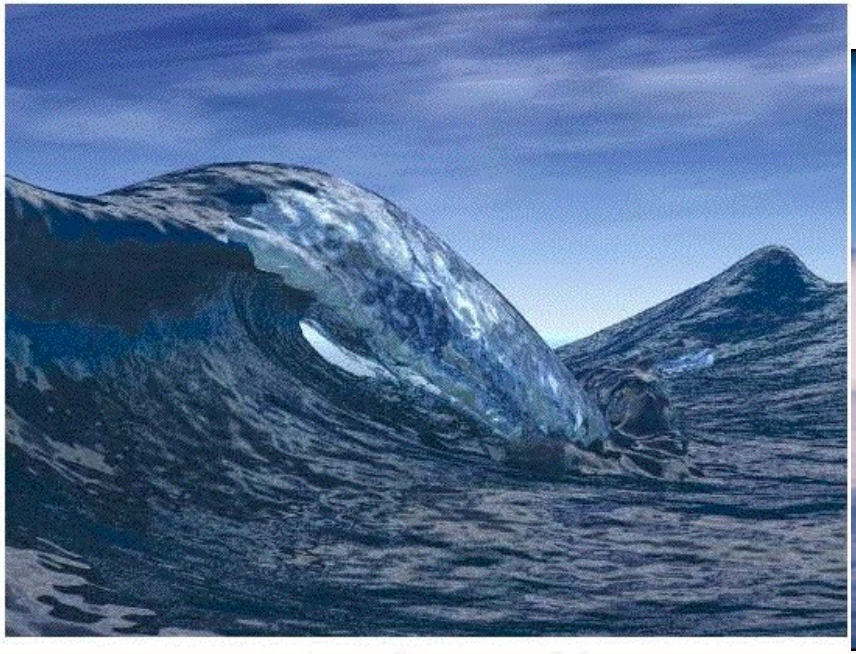
- Fine combination of several fields of Applied Mathematics: Analysis, Partial Differential Equations, Numerical Analysis and Control Theory.

A challenge:

- **Overcome the fact that humans, through mathematical analysis, and computers do not "see" the same reality. Spurious numerical solutions become an obstacle to develop efficient control strategies in real applications.**



Nature versus computer reality:



A few snapshots

Pb1 A simple decomposition in a complex basis

Pb2 Control systems and vibrations

Pb3 Shapes in aeronautics

A simple decomposition in a complex basis

Complex structures often obey a simple pattern. It suffices to look at them in the appropriate perspective and frame, to see better.

A mathematical version: Joint work with J. Duoandikoetxea, 1992.

Theorem: *If* $f \in L^1(R^N, |x|^{k+1})$, *then*

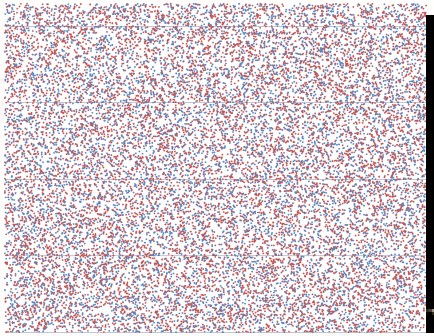
$$f(x) = \sum_{|\alpha| \leq k} \frac{(-1)^{|\alpha|}}{\alpha!} \int f(x) x^\alpha dx \, D^\alpha \delta + \sum_{|\alpha| = k+1} D^\alpha F^\alpha.$$

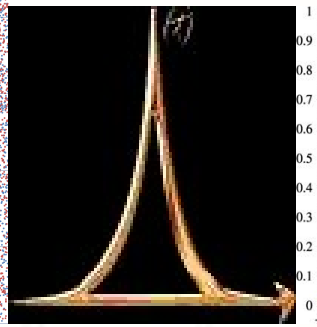
According the solutions of the heat equation obey the asymptotic expansion as $t \to \infty$*:*

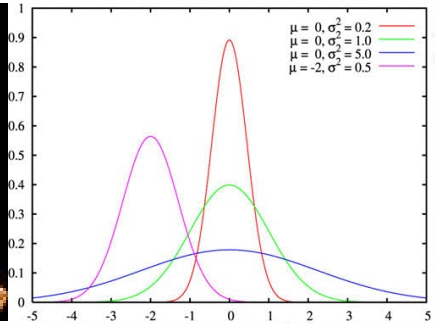
$$u(t) \sim \sum_{|\alpha| \leq k} \frac{(-1)^{|\alpha|}}{\alpha!} \int u_0(x) x^\alpha dx \, D^\alpha G(x,t)$$

where $G = G(x,t)$ *is the gaussian heat kernel:*

$$G(x,t) = (4\pi t)^{-N/2} \exp(-|x|^2/4t).$$







Control systems and vibrations

Control and optimization develop the mathematical theory required to respond to the need, in many applications, of **actuating on systems to perform better**: Noise reduction, scheduling, trajectories of vehicles, irrigation networks, ...

The problem of controllability: Drive the trajectories of an evolution process to a desired final state configuartion.

To find $u \in^m$ such that the solution of

$$\begin{cases} x'(t) = Ax(t) + Bu(t) \\ x(0) = x_0 \end{cases} \tag{1}$$

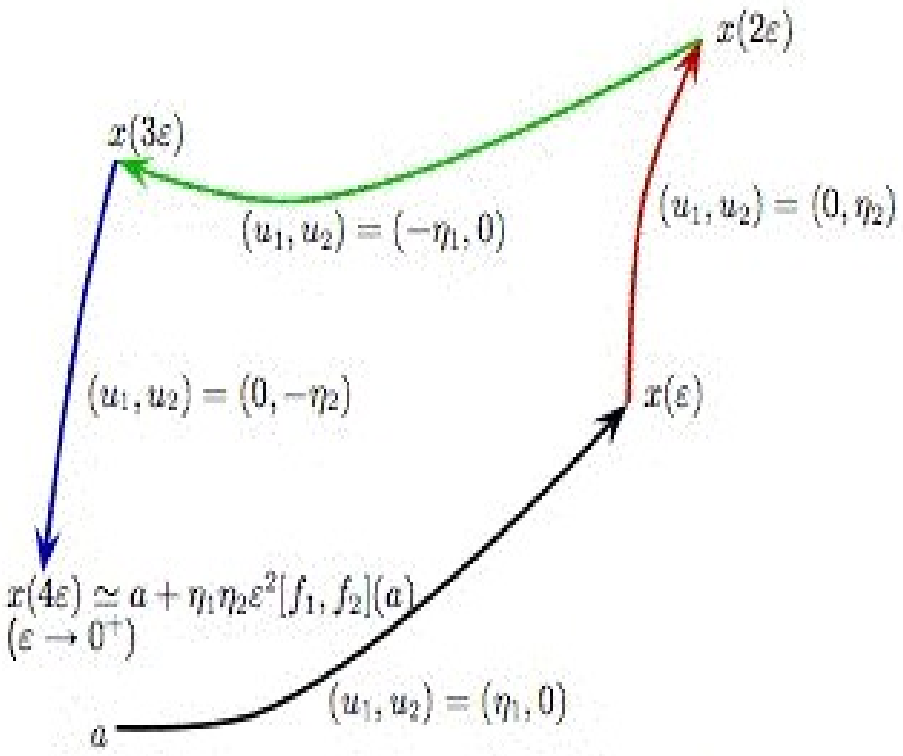
reaches the final target

$$x(T) = x_T.$$

The function $x(t) : [0, T] \rightarrow^n$ is the state and $u(t) : [0, T] \rightarrow^m$ - the control.

Theorem: [*Sukaldariaren Teorema*; Kalman rank condition] The system is controllable if and only if

$$\text{rank}[B, AB, \cdots, A^{n-1}B] = n. \tag{2}$$



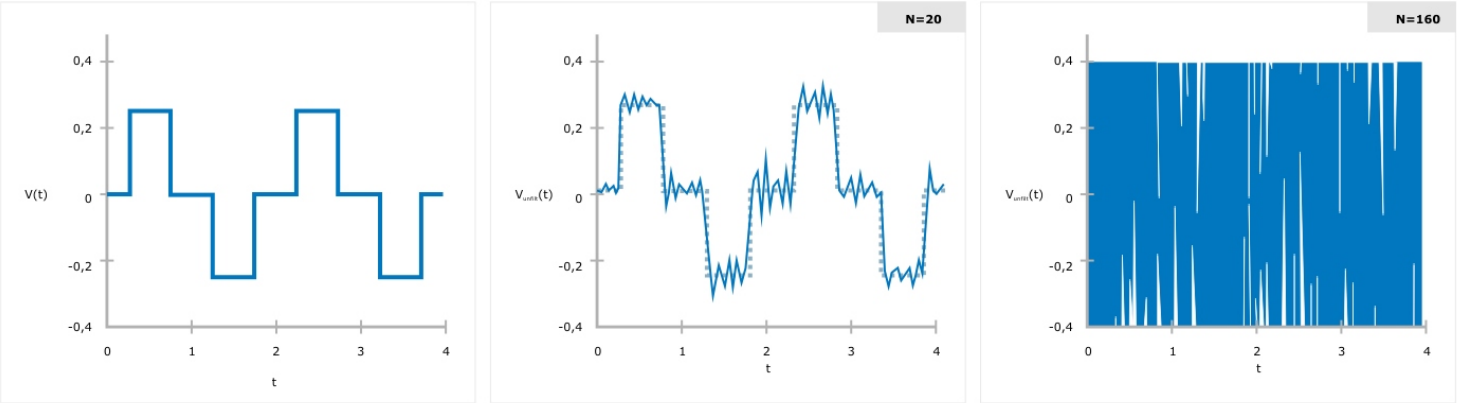
A well known example: Parking and unparking the car.

Cybernetics, Norbert Wiener (1894–1964): the science of communication and control in machines and humans.

Duality: Comunication and control - Sensors and actuators.

When putting together PDE, Numerics and Control, algorithms may diverge and **mathematical analysis** is needed to bring light.

Numerics + control $\neq$ control + numerics



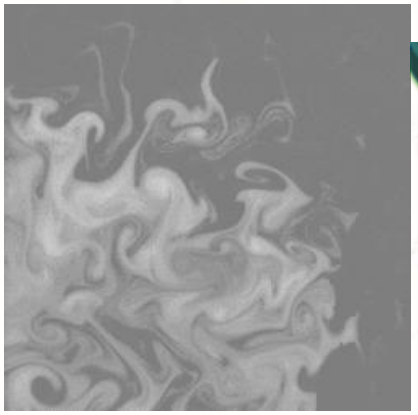
As the dimension of the discretization increases, the computed controls become more and more chaotic and eventually diverge.

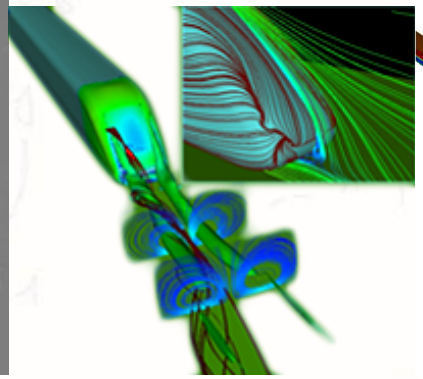
E. Zuazua, SIAM Review, 2005: PDE, Numerical and Control tools have to be tunned finely to guarantee the efficiency of the overall algorithms.

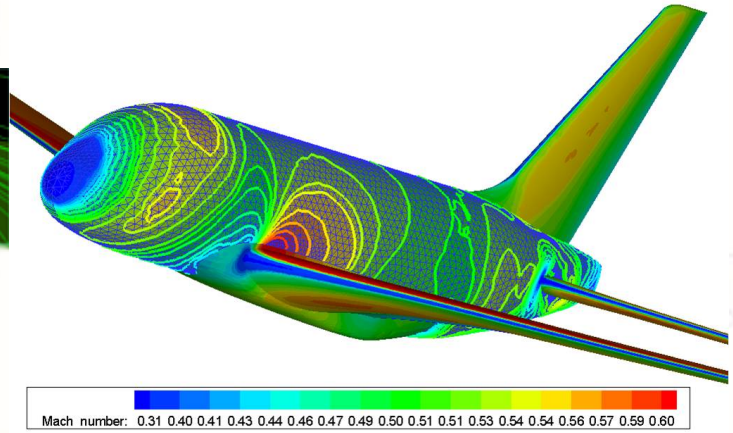
Shapes in aeronautics

Applied Mathematicians address the **inner questions of Mathematics** such as the uniqueness and regularity of solutions of the Navier-Stokes equations in 3 space dimensions (air, water, blood,...) that was identified by the Clay Foundation as one of the Millenium Problems.

http://www.claymath.org/millennium/Navier-Stokes_Equations/







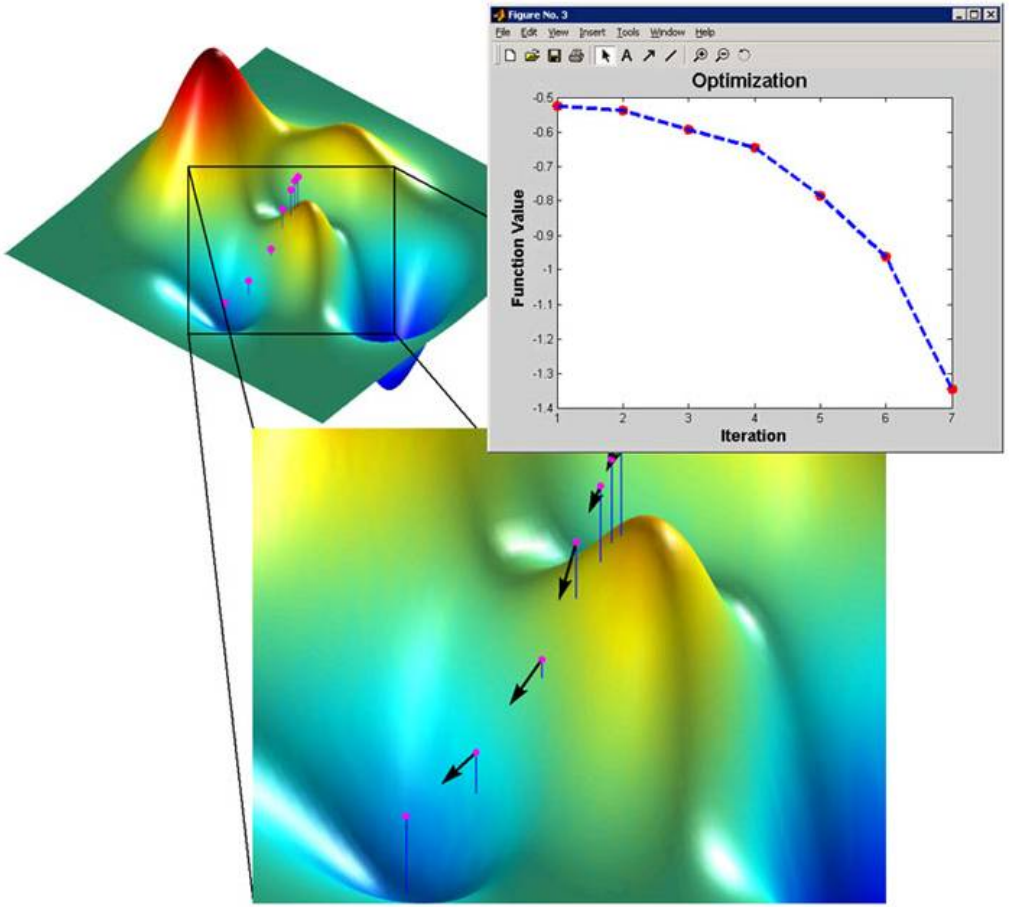
Incompressible N-S equations:

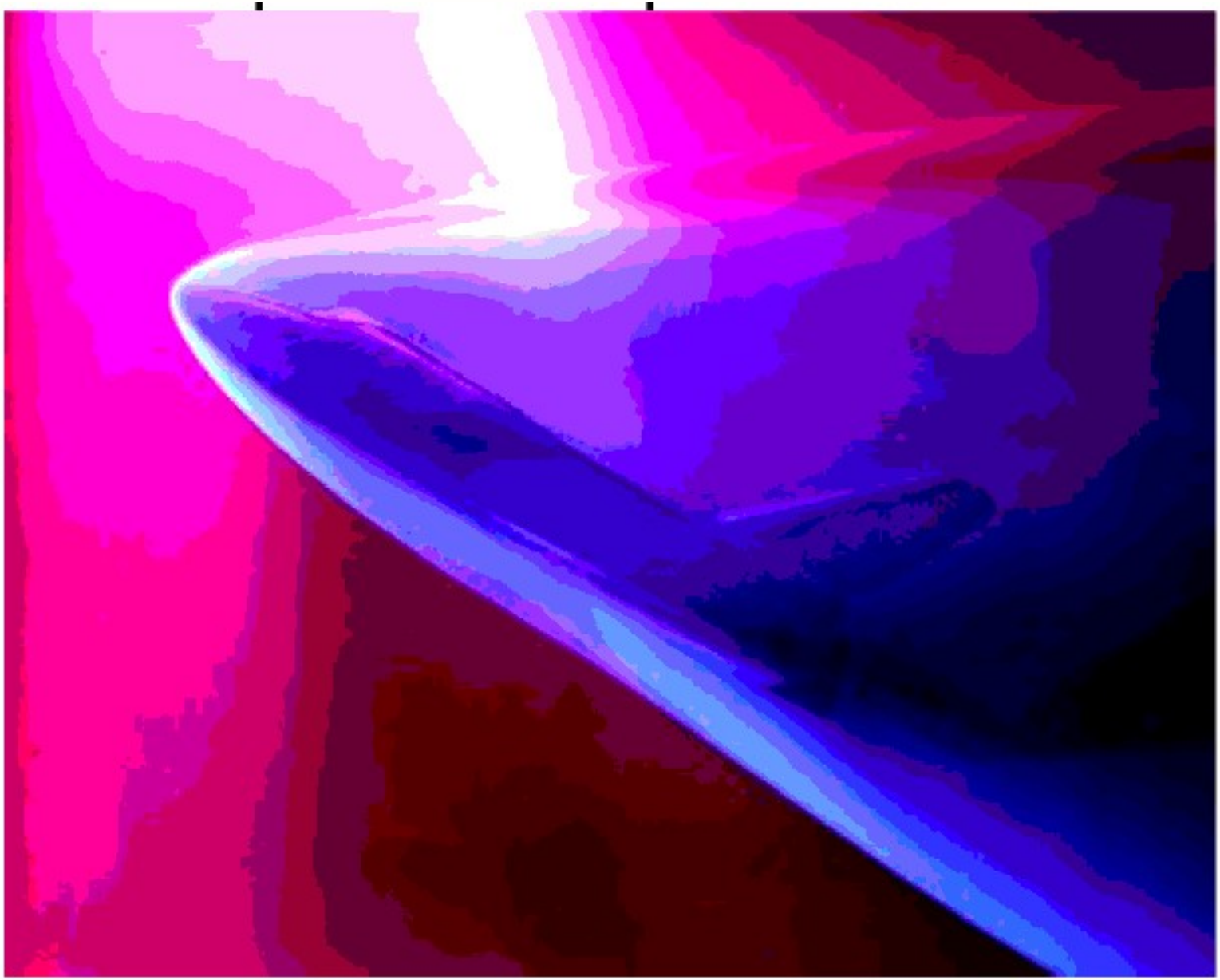
$$u_t - \mu \Delta u + u \cdot \nabla u = \nabla p,$$

$$\mathrm{div} u = 0.$$

But it also addresses other most practical but still challenging issues, such as the development of **efficient numerical methods for aeronautics design.**

Searching for the optimum: Gradient methods

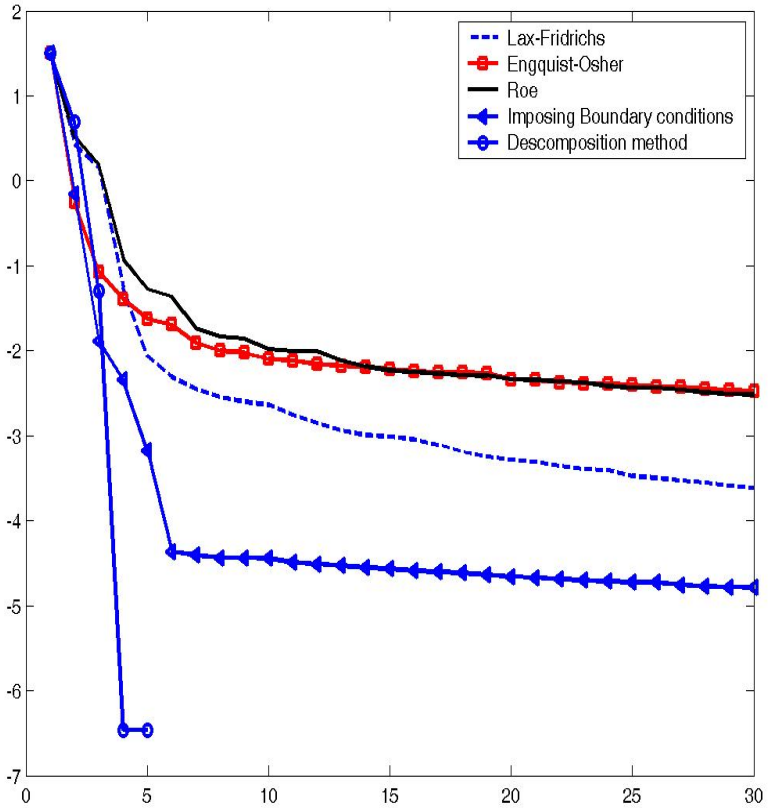


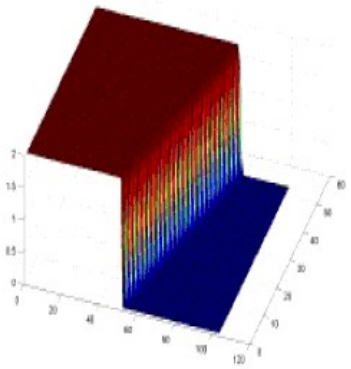


NASA: Visible shocks at the nose in the windtunnel test

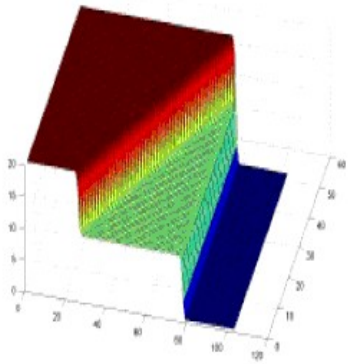
Joint work with C. Castro (UPM) and F. Palacios (AIRBUS-E), E. Zuazua, M3AS, 2008.

The alternating descent method: Takes advantage of the possible singularities on solutions. Descent is computed distinguishing smooth and singular components.





solution u



adjoint p

Milesker!