basque center for applied **mathematics**

ENRIQUE ZUAZUA Jauna
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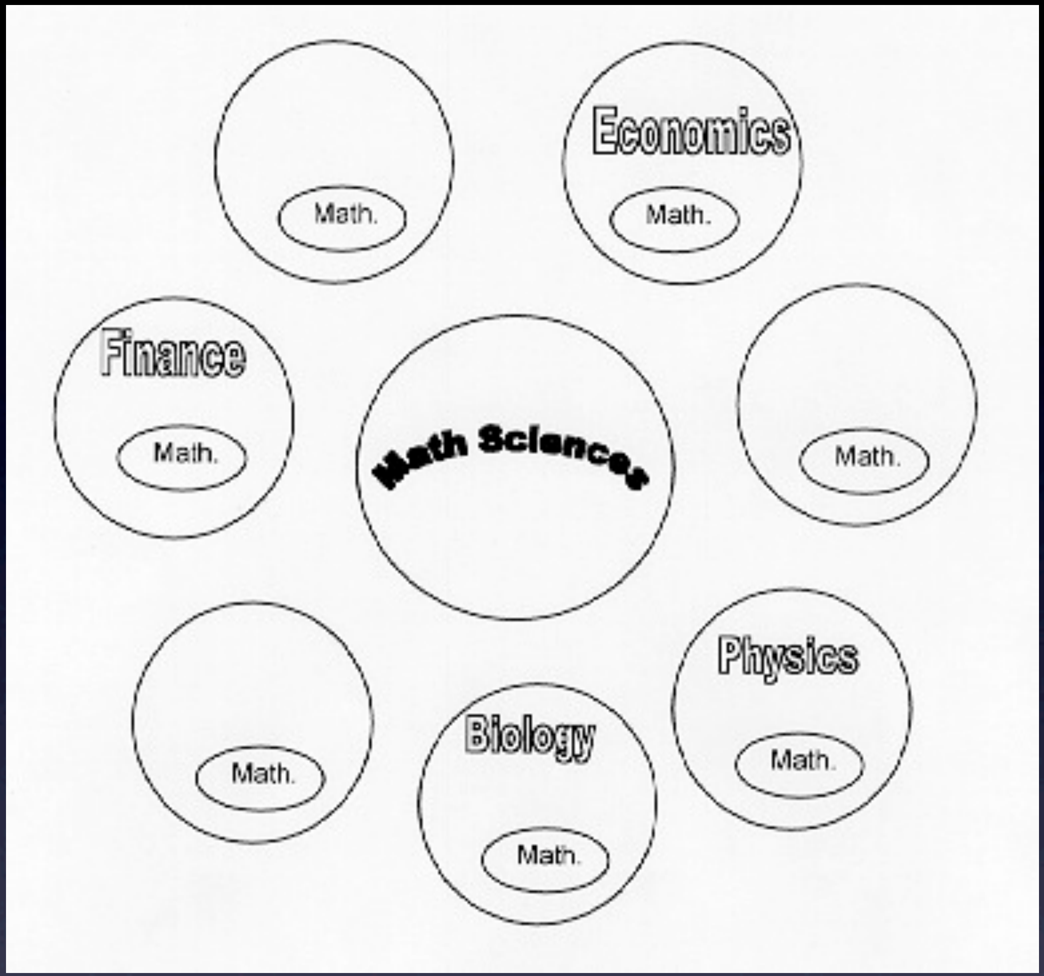

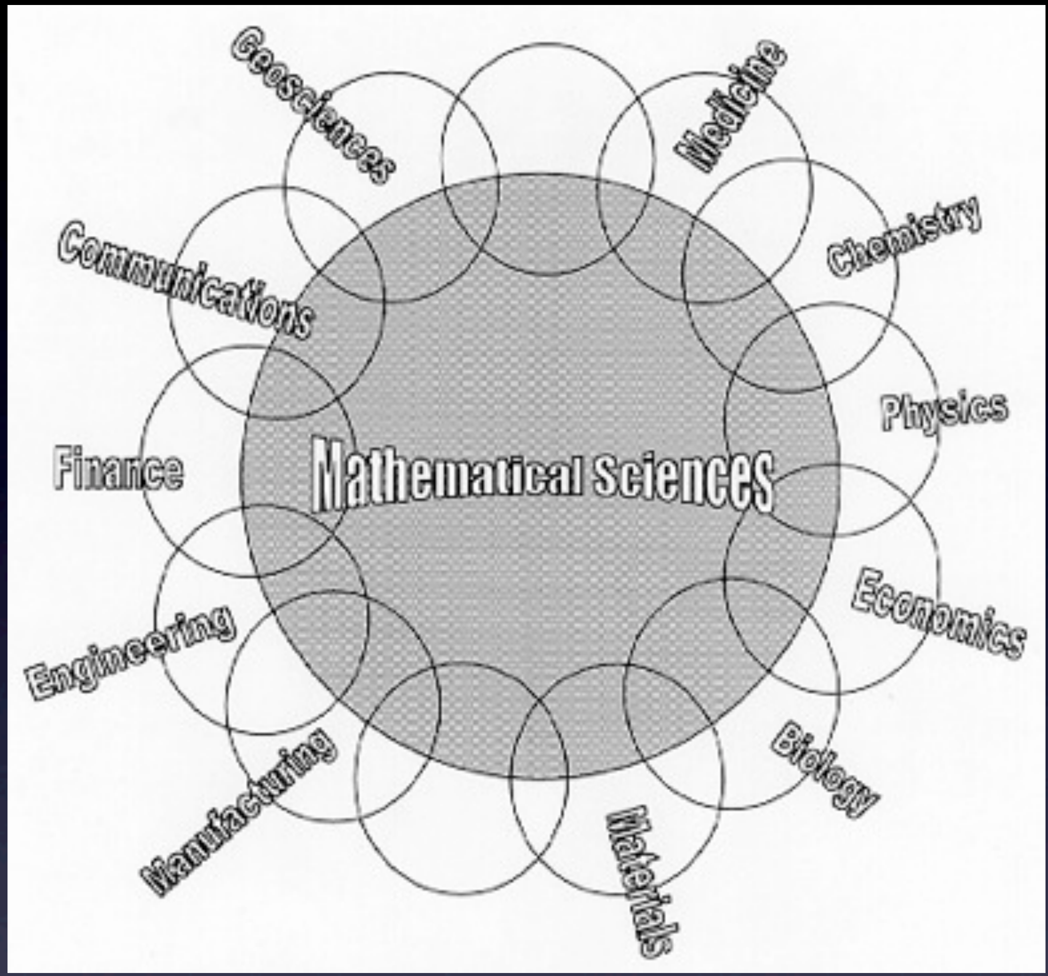

Report of the assesment panel of the U.S. Mathematical Sciences NSF, March 1998. W.E. Odom Lieutenant General, USA, Retired

Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)
Vol. XXVI (98), pp. 605-621

Exact Boundary Controllability of Galerkin's Approximations of Navier-Stokes Equations

JACQUES-LOUIS LIONS – ENRIQUE ZUAZUA*

Abstract. We consider the 2-d and 3-d Navier - Stokes equations in a bounded smooth domain with a boundary control acting on the system through the Navier slip boundary conditions. We introduce a finite-dimensional Galerkin approximation of this system. Under suitable assumptions on the Galerkin basis we prove that this Galerkin approximation is exactly controllable. Moreover we prove that the cost of controlling is independent of the presence of the nonlinearity on the system. Our assumptions on the Galerkin basis are related to the linear independence of suitable traces of its elements over the boundary. At this respect, the one-dimensional Burgers equation provides a particularly degenerate example that we study in detail. In this case we prove local controllability results.

Mathematics Subject Classification (1991): 93B05 (primary), 35Q30, 65M60 (secondary).

1. – Introduction

In a bounded domain Ω of $\mathbb{R}^3$ (we can consider the 2-dimensional case as well) we consider a flow governed by the Navier-Stokes equations. If $y = y(x, t)$ denotes the velocity of the flow and $p = p(x, t)$ the pressure (defined up to a function of time), they satisfy the Navier-Stokes equations

$$(1.1) \qquad \begin{cases} y_t + y \cdot \nabla y - \mu \Delta y = -\nabla p & \text{in } \Omega \times (0, T) \\ \operatorname{div} y = 0 & \text{in } \Omega \times (0, T) \end{cases}$$

where $\mu > 0$ denotes the viscosity and $T > 0$ a given value of time.

We assume that we act on the flow through a *boundary control*. Let $\tau^j = (\tau_1^j, \tau_2^j, \tau_3^j) : \Gamma = \partial\Omega \to \mathbb{R}^3$, $j = 1, 2$ be two smooth vector fields constituting an orthonormal basis of the tangent plane to Ω at each $x \in \Gamma$. Let

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Vol. 47, No. 2, pp. 197–243

Propagation, Observation, and Control of Waves Approximated by Finite Difference Methods*

Enrique Zuazua†

Abstract. This paper surveys several topics related to the observation and control of wave propagation phenomena modeled by finite difference methods. The main focus is on the property of *observability*, corresponding to the question of whether the total energy of solutions can be estimated from partial measurements on a subregion of the domain or boundary. The mathematically equivalent property of *controllability* corresponds to the question of whether wave propagation behavior can be controlled using forcing terms on that subregion, as is often desired in engineering applications. Observability/controllability of the continuous wave equation is well understood for the scalar linear constant coefficient case that is the focus of this paper. However, when the wave equation is discretized by finite difference methods, the control for the discretized model does not necessarily yield a good approximation to the control for the original continuous problem. In other words, the classical convergence (consistency + stability) property of a numerical scheme does not suffice to guarantee its suitability for providing good approximations to the controls that might be needed in applications. Observability/controllability may be lost under numerical discretization as the mesh size tends to zero due to the existence of high-frequency spurious solutions for which the group velocity vanishes. This phenomenon is analyzed and several remedies are suggested, including filtering, Tychonoff regularization, multigrid methods, and mixed finite element methods.

We also briefly discuss these issues for the heat, beam, and Schrödinger equations to illustrate that diffusive and dispersive effects may help to retain the observability/controllability properties at the discrete level. We conclude with a list of open problems and future subjects for research.

Key words. waves, finite difference approximation, propagation, observation, control, heat and beam equations

AMS subject classifications. 65M06, 35L05, 93B07, 93B05

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1. Introduction. This article analyzes numerical methods for approximating the controllability and observability of wave-like equations. These properties can be summarized by the following questions:

- *Observability.* Can waves satisfying a wave equation and suitable boundary conditions be fully reconstructed from measurements on a subregion of the domain or boundary during a given time interval? More precisely, we will

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http://www.siam.org/journals/sirev/47-2/43286.html

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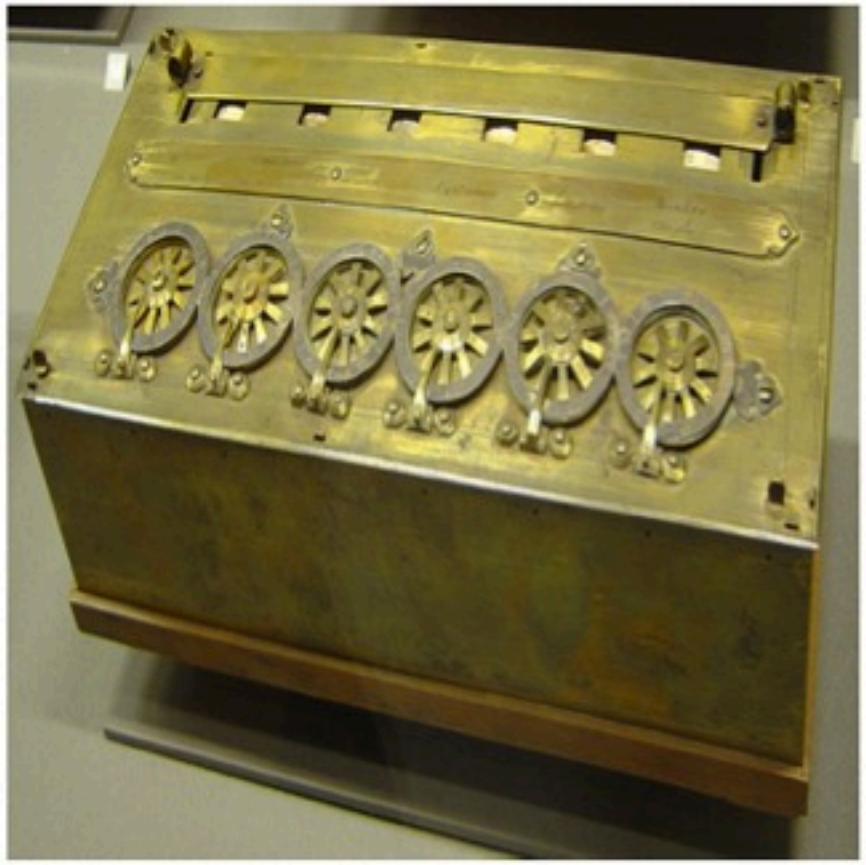
2006ko XI. Euskadi Ikerketa Saria

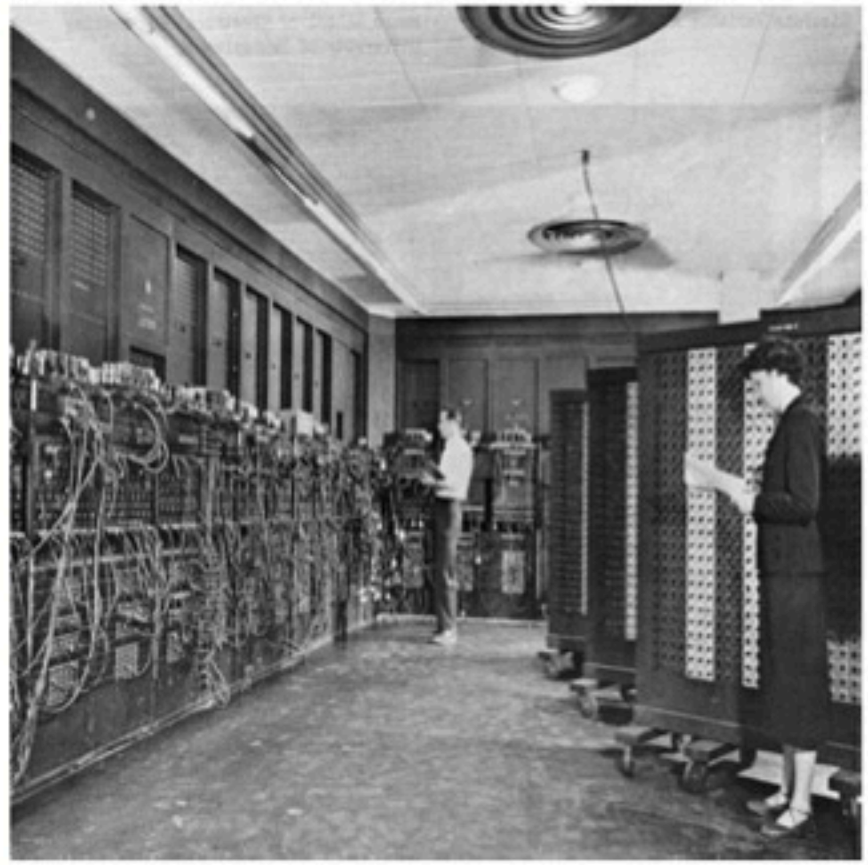
XI Premio Euskadi de Investigación 2006

Lehendakaritza, Vitoria-Gasteiz, Martxoak 22, 2007

Isaac Newton
1642 - 1727

David Hilbert
1862 - 1943

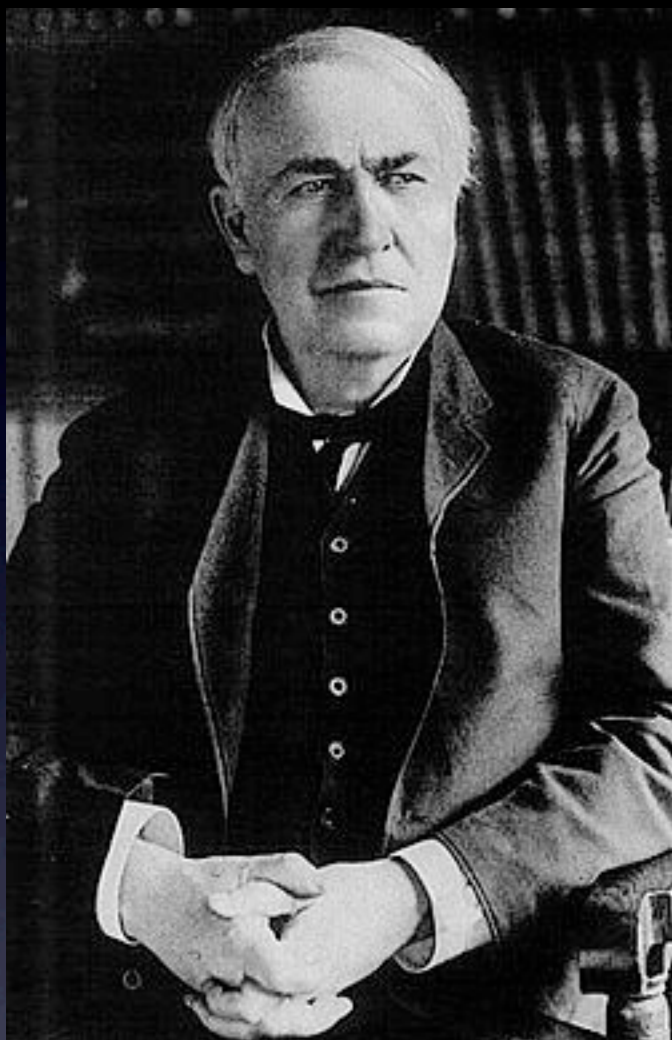




Pascalina
Blaise Pascal, 1645

ENIAC
Electronic Numerical Integrator and Computer, 1946

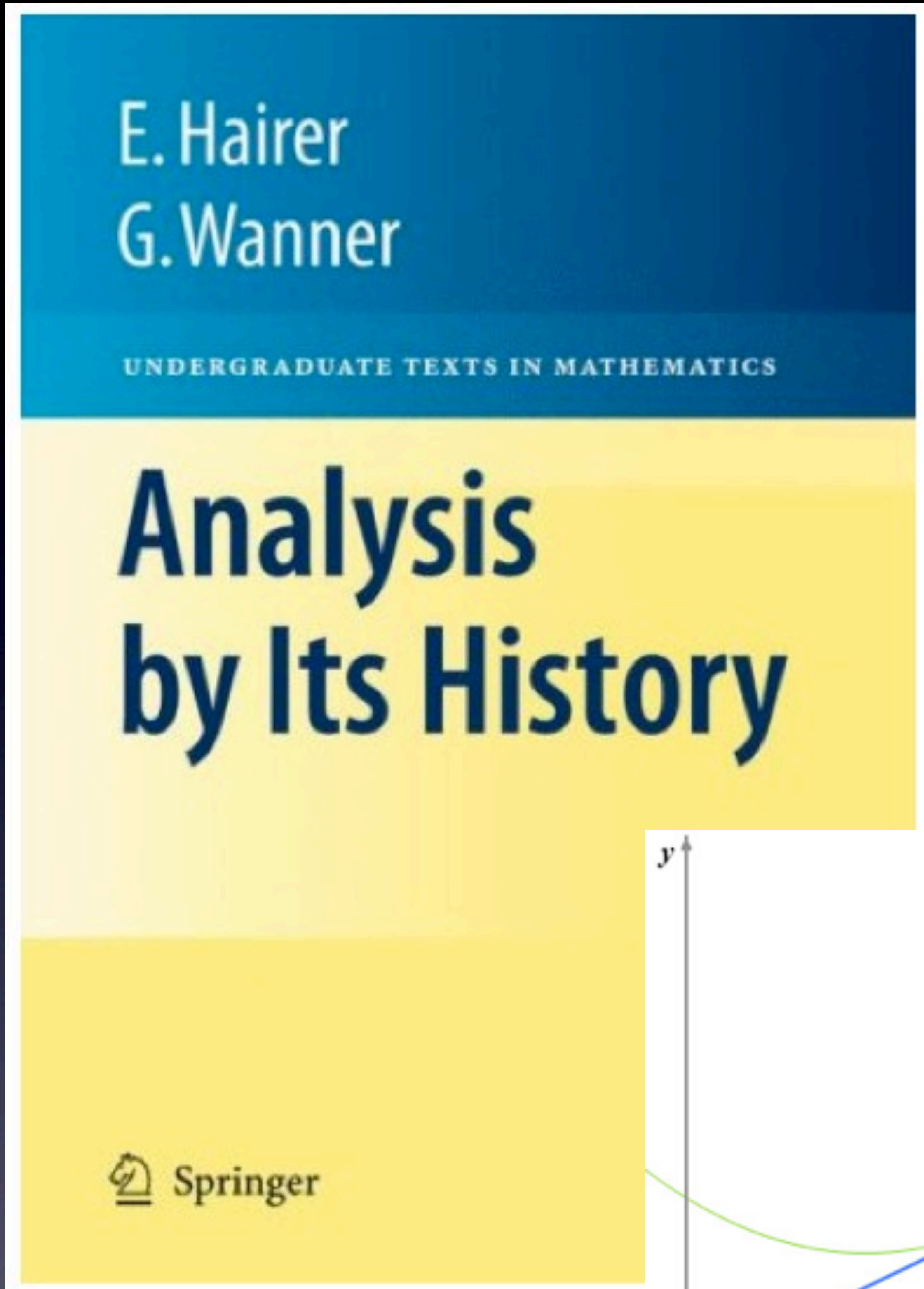
Macbook air
Apple inc, 2008



El genio es uno por ciento de inspiración y un noventa y nueve por ciento transpiración

Thomas A. Edison (1847 - 1931)

Control in an information rich World,
SIAM, R. Murray Ed. 2003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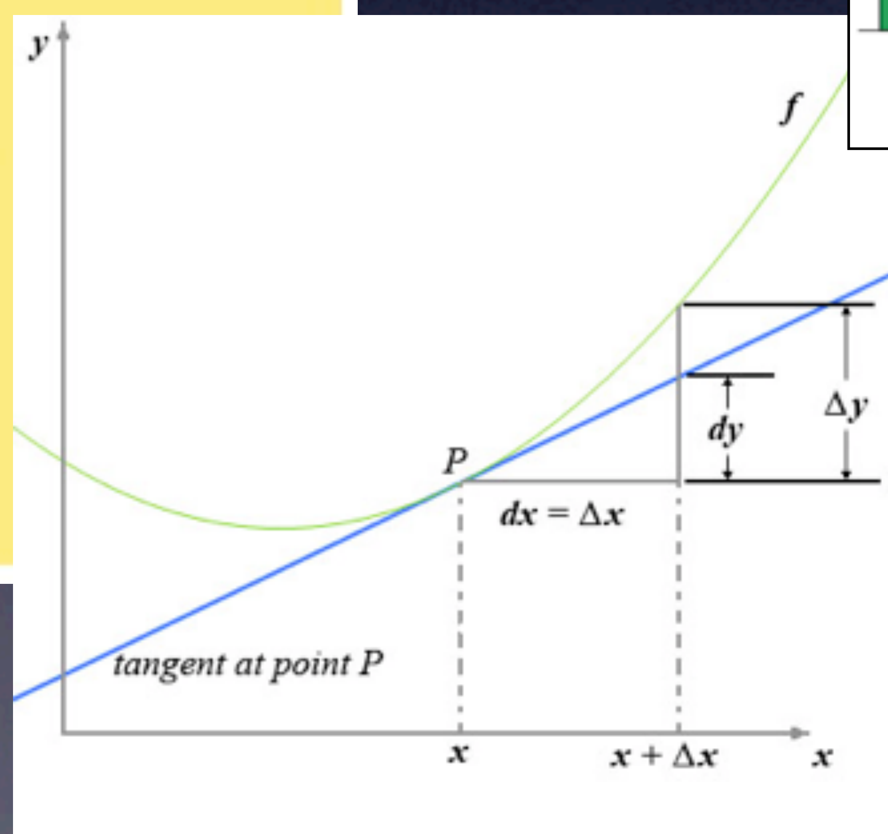
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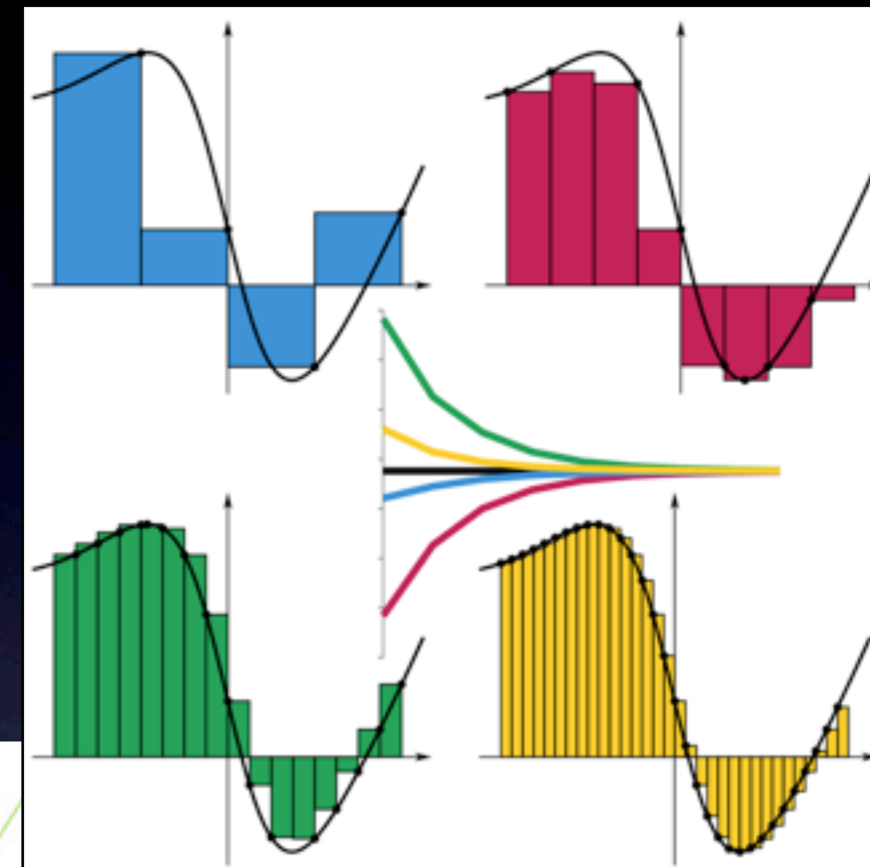

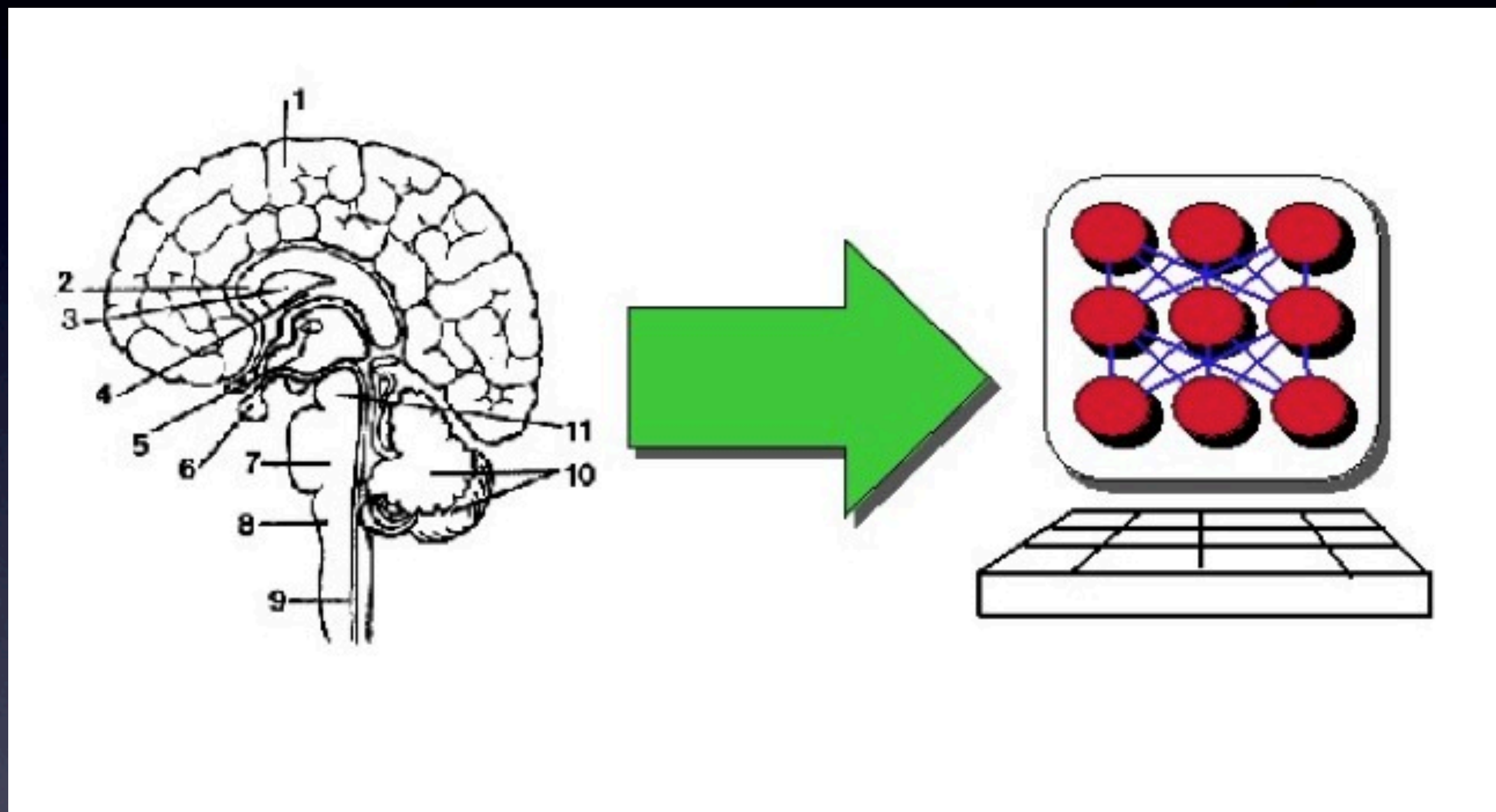
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ENRIQUE ZUAZUA

DIRECTOR CIENTÍFICO DEL BASQUE CENTER FOR APPLIED MATHEMATICS

“El nivel de paro en los jóvenes matemáticos doctores es nulo”

DEIA, Politika,
Miércoles, 9 de noviembre de 2011,
página 10

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First Clay Mathematics Institute Millennium Prize Announced

Prize for Resolution of the Poincaré Conjecture Awarded to Dr. Grigoriy Perelman

March 18, 2010. The Clay Mathematics Institute (CMI) announces today that Dr. Grigoriy Perelman of St. Petersburg, Russia, is the recipient of the Millennium Prize for resolution of the Poincaré conjecture. The citation for the award reads:

The Clay Mathematics Institute hereby awards the Millennium Prize for resolution of the Poincaré conjecture to Grigoriy Perelman.

More ...

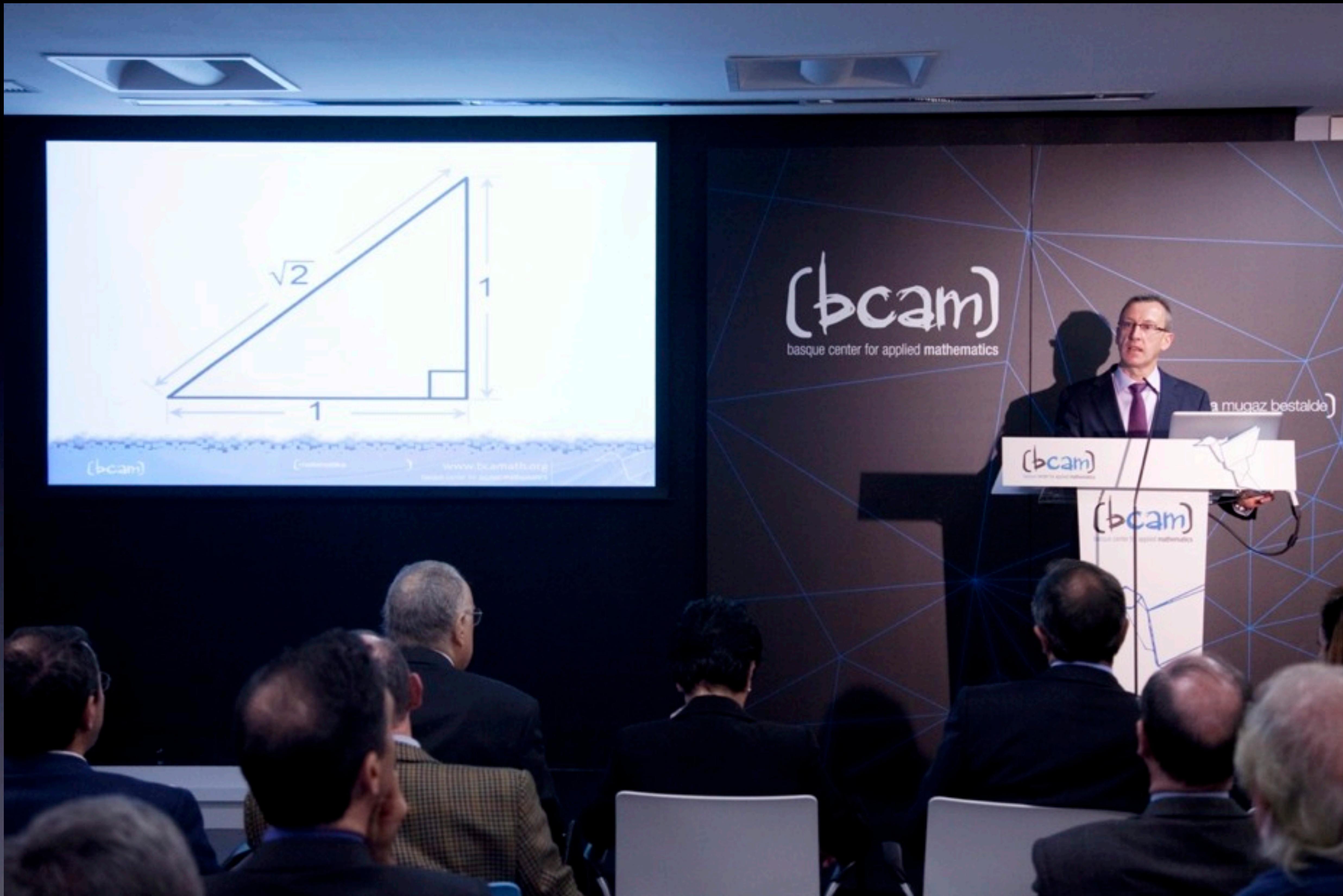
The Millennium Prize Problems

- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

- Rules
- Millennium Meeting Videos

John von Neumann
1903 - 1957

Jacques-Louis Lions
1928 - 2001



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FUTURAMA
MATT GROENING

Entre guiños al público de culto y las Matemáticas está la constante referencia al número **1729**, el "**Taxicab number**"

Una de las veces que **Hardy** (Godfrey Harold Hardy (1877-1947)) fue a visitar a **Ramanujan** (Srinivasa Aaiyangar Ramanujan (1887-1920)) al hospital cuando éste estaba muriéndose. Por hablar de algo le comentó que había venido en un taxi con un número muy aburrido.

Y quué número es ese?, le preguntó Ramanujan.

El 1729 le contestó Hardy.

!Pero cómo puedes decir que ese número es aburrido
si es el menor entero que se puede escribir de dos maneras diferentes como suma de dos cubos!, exclamó Ramanujan.

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

Enseñar y aprender Matemáticas

Roberto Rodríguez del Río Prof. de Matemáticas de Secundaria I.E.S. Gabriel García Márquez. Leganés (Madrid) Prof. Asociado de Matemática Aplicada Universidad Complutense de Madrid Rr_delRio@Mat.UCM.es	Enrique Zuazua Iriondo Catedrático de Matemática Aplicada Universidad Autónoma de Madrid enrique.zuazua@uam.es

La Naturaleza nada hace en vano,
y más es en vano cuanto a menos sirve;
pues la Naturaleza se complace en la simplicidad,
y no adopta la pompa de las causas superfluas.
Isaac Newton, Principia.

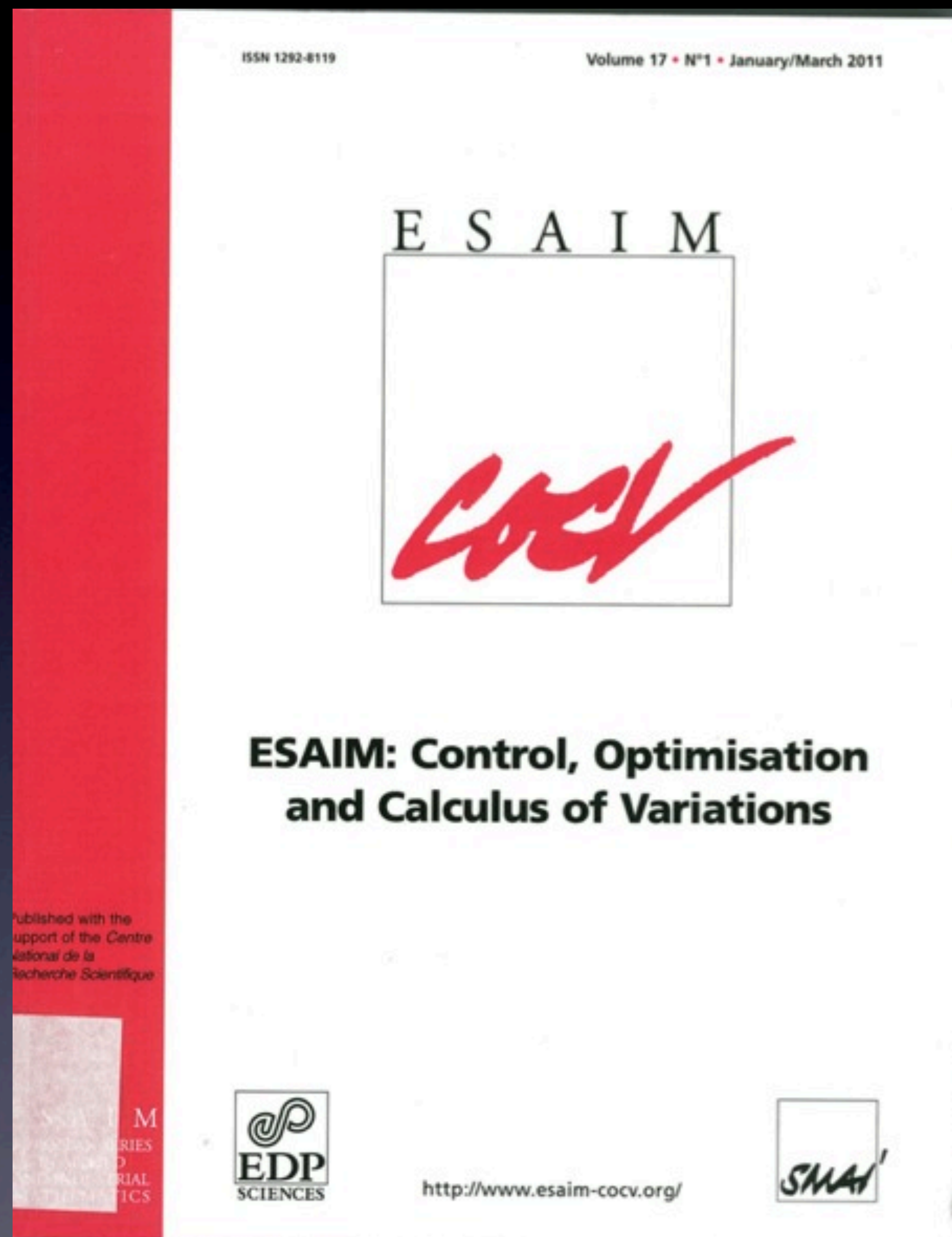
1. Introducción: algunas claves para un importante debate

A pesar de que el deseo de muchos matemáticos y profesores de Matemáticas sea otro, las Matemáticas no se encuentran entre las preocupaciones más importantes del ciudadano. Sin embargo, son pocos los que a lo largo de su vida no han tenido, en algún que otro momento, contacto con ellas. Y prácticamente todo el mundo está de acuerdo en que es necesario un conocimiento básico de las Matemáticas para desenvolverse con una cierta soltura en la vida cotidiana. Por otra parte, si hay alguna materia que en las escuelas levanta pasiones, y también grandes desafecciones, esta es precisamente la de Matemáticas.

Las Matemáticas son ya una Ciencia antigua. Existen desde mucho antes de que se le dieran nombre y sus orígenes se remontan al menos al momento en que el ser humano empieza a contar. Cabría también decir, como en su momento afirmó Galileo, que el Universo está escrito en lenguaje matemático[1] y de ese modo estableceríamos que las Matemáticas surgen con nuestro Universo, de manera simultánea. Sin remontarnos tan lejos en el tiempo, Albert Einstein se preguntaba a principios del siglo que acabamos de dejar: "*¿cómo es posible que la matemática, un producto del pensamiento humano independiente de la experiencia, se adapte tan admirablemente a los objetos de la realidad?*"

---

[1] "*La filosofía está escrita en ese grandísimo libro abierto ante los ojos; quiero decir, el Universo, pero no se puede entender si antes no se aprende a entender la lengua, a conocer los caracteres en los que está escrito. Está escrito en lengua matemática y sus caracteres son triángulos, círculos y otras figuras geométricas, sin las cuales es imposible entender ni una palabra; sin ellos es como girar vanamente en un oscuro laberinto.*" Galileo Galilei. El Ensayador.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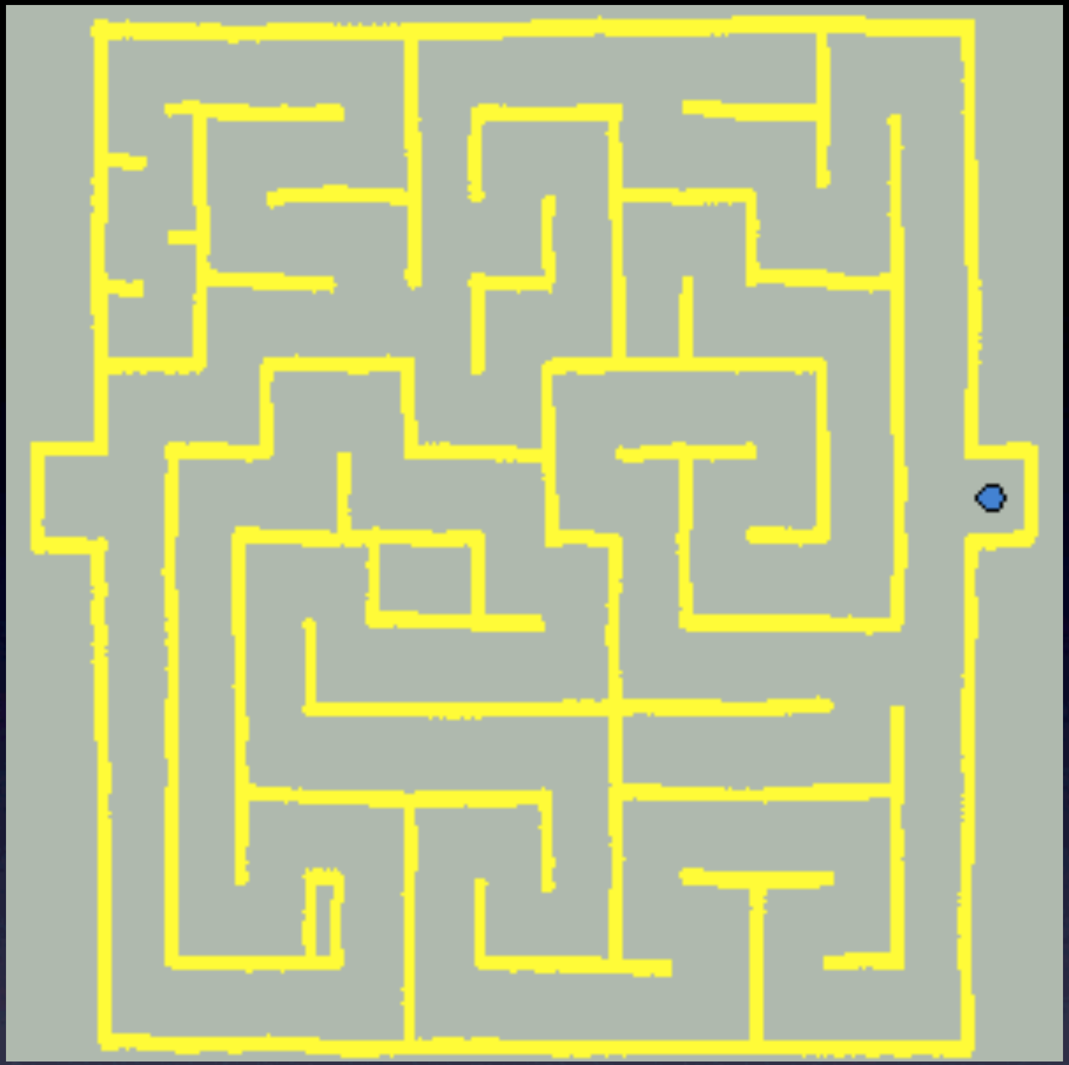
SMAI

Cédric Villani (1973 -) is being awarded the 2010 Fields Medal for his proofs of nonlinear Landau damping and convergence to equilibrium for the Boltzmann equation.

Ezagutzen duguna ur tanta da. Ezagutzen ez duguna, ordea, ozeanoa....

Lo que sabemos es una gota de agua; lo que ignoramos es el océano....

Ramón Escobedo



Ramón Escobedo