

# A Parallel Meta-Heuristic for Solving a Multiple Asymmetric Traveling Salesman Problem with Simultaneous Pickup and Delivery modeling Demand Responsive Transport Problems

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**Abstract.** Transportation is an essential area in the nowadays society. Due to the rapid technological progress, it has gained a great importance, both for business sector and citizenry. Among the different types of transport, one that has gained notoriety recently is the transportation on-demand, because it can affect very positively the people quality of life. There are different kinds of on-demand transportation systems, being the Demand Responsive Transit (DRT) one of the most important one. In this work, a real-life DRT problem is proposed, and modeled as a Rich Traveling Salesman Problem. Specifically, the problem presented is a Multiple Asymmetric Traveling Salesman Problem with Simultaneous Pickup and Delivery. Furthermore, a benchmark for this new problem is also proposed, and its first resolution is offered. For the resolution of this benchmark the recently developed Golden Ball meta-heuristic has been implemented.

**Keywords:** On demand transportation, Demand Responsive Transport, Traveling Salesman Problem, Golden Ball, Meta-heuristic, Combinatorial Optimization.

## 1 Introduction

Transportation is an important issue for the society these days, both for citizens and the business sector. Regarding the transportation in the business world, the rapid advance of technology has made the logistic increasingly important in this area. The fact that anyone in the world can be well connected has led transport networks to be very demanding, something that was less important in the past. Today, a competitive logistic network can make the difference between some companies and others.

On the other hand, public transport is used by almost all the population and it affects the life quality of the people. In addition, there are different kinds of public transportation systems, each one with its own characteristics.

Nonetheless, all of them share the same disadvantages, which are the finite capacity of the vehicles, the geographical area of coverage, and the service schedules and frequencies.

With the intention of addressing these drawbacks the concept of Transportation-On-Demand (TOD) arises [1]. This concept is related with the transportation of goods or passengers between specific origins and destinations at the request of customers. Almost all the TOD systems are characterized by sharing the following three conflicting objectives: minimizing operating costs, maximizing the number of requests served, and minimizing clients inconveniences.

There are several kind of TOD problems, being the Demand Responsive Transport, or Demand Responsive Transit (DRT) one of the most important [2]. This problem is characterized by flexible routing and scheduling of small/medium vehicles operating in shared-ride mode between pick-up and drop-off locations according to passengers needs. A DRT system can be applicable in situations where passengers are transported between concrete origins and destinations. One common application of these kinds of systems is the transport service in areas of low passenger demand, where a regular transport service is not economically viable. Another typical application is the door-to-door services for handicapped or elderly people. In this context, users formulate two different related requests: an outbound request from home to a destination, and an inbound request for the return trip. This kind of transport has a great social interest since, above all, it helps to ensure welfare of people with special needs.

DRT and other kind of on-demand problems are the focus of many studies nowadays [3,4]. In addition, many sophisticated on-demand systems have been implemented in several major cities across the world, as Bristol (United Kingdom)<sup>1</sup>, Cape Town<sup>2</sup>, or London<sup>3</sup>.

The objective of this research is to address one DRT problem. To achieve this goal, the DRT problem has been modeled as a Rich Traveling Salesman Problem (R-TSP), also known as Multi-Attribute Traveling Salesman Problem. Nowadays, these rich or multi-attribute problems, as well as the multi-attribute vehicle routing ones, are a hot issue in the literature [5]. These sorts of problems are specific cases of routing problems, with complex formulations and multiple restrictions. Furthermore, they have a great scientific interest because of their complexity of resolution, which represents a scientific challenge, and their applicability to real-world situations, which is greater than the conventional routing problems.

In this work an R-TSP is presented, to be more accurate, a Multiple Asymmetric Traveling Salesman Problem with Simultaneous Pickup and Delivery. Furthermore, the first benchmark for this problem is also detailed in this paper, and its first ever resolution is offered. To deal with this problem the recently proposed Golden Ball (GB) meta-heuristic has been implemented [6].

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<sup>1</sup> <http://www.bristololdiaride.org.uk/>

<sup>2</sup> <https://www.capetown.gov.za/en/Transport/Pages/AboutDialARide.aspx>

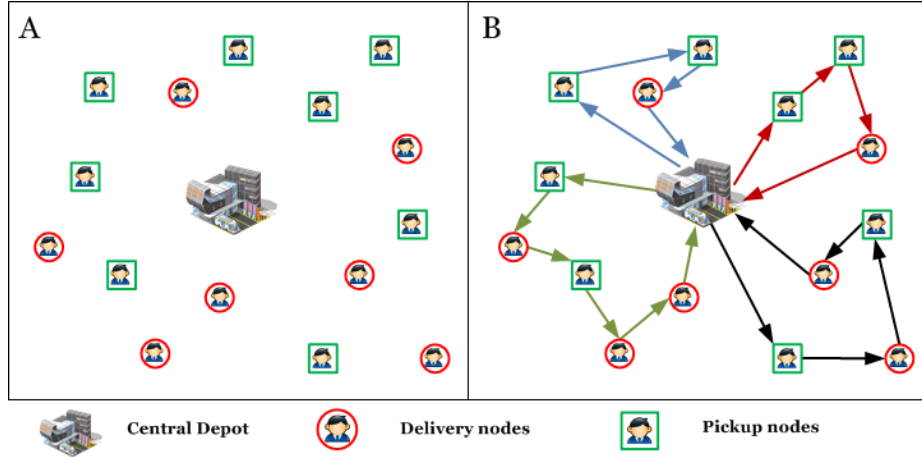
<sup>3</sup> <https://www.tfl.gov.uk/modes/dial-a-ride/>

The remainder of this work is structured as follows: In the following section the proposed MA-TSP-SPD is described and formulated. In Section 3 the benchmark used for the presented problem is detailed. In Section 4 the technique implemented for the resolution of this benchmark is depicted. Additionally, the experimentation carried out is described in Section 5. Finally, conclusions and future work are explained in Section 6.

## 2 Description of the proposed MA-TSP-SPD

As has been introduced in the previous section, an R-TSP is proposed in this paper, with the aim of addressing different kind of DRT problems. The principal feature of a R-TSP problem is its complex formulation, which is composed by multiple constraints. This feature directly leads to an increased complexity of resolution, which entails to a major scientific challenge at the same time. DRT problems are important because they model many real world problems and, therefore, efficient solving techniques for that kind of problems can be useful in many interesting practical applications. The problem presented in this research is a MA-TSP-SPD, which has three main characteristics.

1. *Multiple Vehicles*: This is a typical feature of the often studied Multiple Traveling Salesman Problem [7]. In this way, a fleet  $K$  composed by a finite and fixed  $k$  number of vehicles is available in the proposed MA-TSP-SPD. These  $k$  vehicles have to be employed to meet the customers needs. Additionally, there is a central depot in which all the vehicle routes have to begin and end. This feature requires the problem to plan exactly  $k$  paths, one for each available vehicle. Besides, each vehicle cannot plan a route composed by more than a fixed  $q$  nodes.
2. *Asymmetry*: The traveling costs in the proposed MA-TSP-SPD are asymmetric. This means that the traveling cost from any  $i$  node to another  $j$  node is different from the reverse trip cost. This feature is not common in most routing problems that can be found in the literature, and it brings realism and complexity to the problem. Anyway, asymmetric costs has been applied previously in the literature [8, 9]. Because of the realism it brings, it noteworthy that this feature is very valuable in DRT situations.
3. *Simultaneous Pickup and Delivery*: This property is an adaptation of the often used pickup and delivery system of some routing problems [10, 11]. Basically, this system consists in the existence of two types of nodes, the *delivery nodes* and the *pickup nodes*. The first ones are those points in where the peoples leave the vehicle. On the other hand, in *pickup nodes* is where the people who have requested the transportation access to the vehicle. In addition, it is important to highlight that, due to the simultaneous nature of this feature, in one concrete node more than one customer can leave or take the vehicle. This fact leads to the generation of routes in which the number of delivery nodes is greater than the amount of pickup nodes, and vice versa. Furthermore, the depot, mimicking the behavior of a central bus station, can also act as a pickup or delivery node.



**Fig. 1.** A 15-noded and  $k=4$  possible instance of the MA-TSP-SPD, and a possible solution

This feature is important in many DRT problems, as for example, the door-to-door transportation of elderly people.

Therefore, the proposed MA-TSP-SPD is a rich routing problem with asymmetric costs, in which the objective is to find exactly  $k$  number of different routes, each one with a maximum length of  $q$  nodes, minimizing the total cost of the complete solution.

In Figure 1(a) an example of a MA-TSP-SPD instance with 15 nodes, and  $k=4$  is depicted. Furthermore, in Figure 1(b) a possible solution for this instance is shown.

In this manner, the presented MA-TSP-SPD can be defined on a complete graph  $G = (V, A)$  where  $V = \{v_0, v_1, v_2, \dots, v_n\}$  is the set of vertexes which represents the nodes of the system. On the other hand,  $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$  is the set of arcs which represents the interconnection between nodes. Each arc has an associated distance cost  $d_{ij}$ . Due to the asymmetry feature  $d_{ij} \neq d_{ji}$ . Furthermore, the vertex  $v_0$  represents the depot, and the rest are the visiting points. In addition, with the aim of facilitating the problem formulation, the set of customers  $V$  can be separated into two different subsets, the first one for the *pickup nodes*  $PN = \{pn_1, pn_2, \dots, pn_n\}$ , and the second one for the *delivery nodes*  $DN = \{dn_{n+1}, dn_{n+2}, \dots, dn_{n+m}\}$ .

Additionally, the permutation codification has been used for the representation of the solutions. Thus, each solution  $X$  is encoded by a permutation of numbers, which represents the different routes that compose that solution. Besides, with the aim of distinguishing the routes in one solution, they are separated by zeros. For example, supposing a set of six *pickup nodes*  $PN = \{pn_1, pn_2, pn_3, pn_4, pn_5, pn_6\}$ , and seven *delivery nodes*  $DN =$

$\{dn_7, dn_8, dn_9, dn_{10}, dn_{11}, dn_{12}, dn_{13}\}$ . One possible solution with  $k=3$  would be  $X = (pn_2, pn_5, dn_7, dn_9, \mathbf{0}, pn_4, dn_{12}, dn_{11}, pn_6, dn_{13}, \mathbf{0}, pn_1, dn_{10}, pn_3, dn_8)$ .

Finally, the proposed MA-TSP-SPD can be mathematically formulated in the following way:

Minimize:

$$\sum_{i=0}^{n+m} \sum_{j=0}^{n+m} \sum_{r=1}^k d_{ij} x_{ij}^r \quad (1)$$

Where:

$$x_{ij}^r \in \{0, 1\}, \quad i, j = 0, \dots, n+m, i \neq j; r = 1 \dots k \quad (2)$$

Subject to constraints:

$$\sum_{i=0}^{n+m} \sum_{r=1}^k x_{ij}^r = 1, \quad i = 0, \dots, n+m; i \neq j \quad (3)$$

$$\sum_{j=0}^{n+m} \sum_{r=1}^k x_{ij}^r = 1, \quad j = 0, \dots, n+m; j \neq i \quad (4)$$

$$\sum_{i=0}^{n+m} \sum_{j=0}^{n+m} x_{ij}^r \leq q, \quad r = 1 \dots k \quad (5)$$

$$\sum_{j=0}^{n+m} \sum_{r=1}^k x_{0j}^r = k \quad (6)$$

$$\sum_{i=0}^{n+m} \sum_{r=1}^k x_{i0}^r = k \quad (7)$$

$$\sum_{i=0}^{n+m} x_{ij}^r - \sum_{l=0}^{n+m} x_{jl}^r, \quad j = 0, \dots, n+m; r = 1 \dots k \quad (8)$$

$$\sum_{j=0}^{n+m} x_{ij}^r - \sum_{l=0}^{n+m} x_{li}^r, \quad i = 0, \dots, n+m; r = 1 \dots k \quad (9)$$

The first clause represents the objective function, which is the sum of the costs of all routes of the solution, and it must be minimized. The formula 2 depicts the nature of the binary variable  $x_{ij}^k$ , which is 1 if the vehicle  $k$  uses the arc  $(i, j)$ , and 0 otherwise. Functions 3 and 4 assure that all the nodes are visited exactly once. Besides, sentence 5 guarantees that all routes are shorter than the maximum allowed length  $q$ . On the other hand, constraints 6 and 7 ensure that the total amount of vehicles leaving the depot, and the number of vehicles that return to it is the same. In addition, that number has to be  $k$ , i.e., the total amount of available vehicles. Finally, the correct flow of each route is ensured thanks to functions 8 and 9 functions.

### 3 Description of the used benchmark

As it is well-known, the use of a benchmark to study how good a technique is at solving an optimization problem is a crucial factor. In this way, the benchmark presented in this work for the proposed MA-TSP-SDP is the same as presented in the work [12], which is a modification of the ATSP Benchmark that can be found in the TSPLib Benchmark [13].

In this way, 19 different instances have been used for the experimentation, which have from 17 to 443 nodes. It is noteworthy that the first node of each instance is the depot. Additionally, a parameter called  $type_i$  suggests if the node  $i$  is a *delivery node* or a *pickup node*. This parameter has been set using the following procedure:

$$type_i = \textit{pickup node}, \quad \forall i \in \{1, 3, 5, \dots, n\}$$

$$type_i = \textit{delivery node}, \quad \forall i \in \{2, 4, 6, \dots, n\}$$

Furthermore, the number of vehicles available for each instance has been established in  $k=4$ . Besides, the maximum length of each route has been set in  $q = n/3$ , where  $n$  is the total number of nodes of the instance.

With the aim of allowing the replication of this experimentation, the benchmark developed is available under request to the corresponding author of this paper.

### 4 The proposed Golden Ball

The problem proposed in this paper is applicable to real-world situations, this is the reason why it has been opted for a meta-heuristic with a great robustness and quick execution. Robustness is the ability of providing always similar results, leading to a small standard deviation. This feature, along with the quick execution, is very appreciated in real-world applications. The algorithm selected to deal with the presented MA-TSP-SPD, the Golden Ball, is a recently proposed technique which meets these two requirements. The first complete version of the GB, and its practical use for solving complex problems have been presented in 2014 by Osaba, Diaz and Onieva [6].

The main characteristics of the GB can be summarized as follows. The GB is a multiple-population based meta-heuristic inspired by some concepts of soccer sport. First, in the initialization phase, the whole population of solutions (called players) is randomly created. Then, these created players are randomly divided among a fixed number of subpopulations (called team). Each team has its own training method (or coach), which is randomly assigned in this first phase. This training is the way in which each player in the team individually evolves along the execution. One training function could be, for example, the well-known 2-opt [14]. Another important training is the called Custom Training. In this training, a player which is trapped in a local optimum receives a special training in cooperation with the best player of its team. One custom training function could be, for example, the well-known Order Crossover [15].

**Algorithm 1:** Pseudocode of the GB algorithm

```
1 Initialization of the initial population;
2 Division of players into different teams;
3 repeat
4   Competition league is restarted for each matchday do
5     for each team  $t_i$  in the system do
6       Training phase for  $t_i$ ;
7       Custom training session for  $t_i$ ;
8       Calculation of the quality of  $t_i$ ;
9     end
10    Matchday in which matches are played;
11  end
12  Period of transfers;
13 until termination criterion reached;
14 Return the fitness of best player of the system;
```

Once the initialization phase is finished, the competition phase starts. This second step is divided in seasons, composed by weeks. Every week all the teams train independently, and they face each other creating a competition league. At the end of every season, a transfer procedure takes place. In this procedure the players and coaches can switch teams. The competition phase is repeated iteratively until the termination criterion is reached.

The execution of the GB is briefly schematized in Algorithm 1. For further information about the GB, the reading of [6] is highly recommended.

## 5 Experimentation

In this section the conducted experimentation is detailed, and it is divided into two different subsections. In the first one (Section 5.1), the parametrization used for the GB is described. On the other hand, in Section 5.2 the obtained results obtained are depicted.

### 5.1 Parameters of the GB

The population size used for the GB is 48, which has been divided into 4 teams of 12 players each. In addition, the number of trainings without improvement needed to perform a custom training and a special transfer are, respectively, 6 and 12. The well-known *2-opt* and *Insertion* functions have been used as conventional training functions. These operators are intra-route functions [16], i.e., they work within a specific route. Additionally, two inter-route functions have been developed:

- *Swapping Routes*: This operator selects randomly two nodes of two randomly selected routes. These nodes are swapped.

Number of teams (TN)	4
Number of players per team (PT)	12
Number of trainings without improvement for a <i>custom training</i>	6
Number of trainings without improvement for a <i>special transfers</i>	12
Conventional training functions	2-opt, Insertion, Swapping Routes, & Insertion Routes
Custom training function	Random Route Crossover

**Table 1.** Summary of the characteristics of *GB*

- *Insertion Routes*: This function selects and extracts one random node from one random route. After that, this node is re-inserted in a random position in another randomly selected route.

It is noteworthy that all these functions take into account both the vehicles capacity, and the class of the nodes demands, never making infeasible solutions. Furthermore, the Random Route Crossover has been used as custom training function [6]. Finally, the configuration used for *GB* is summarized in Table 1.

## 5.2 Results

All the tests have been performed on an Intel Core i5 2410 laptop, with 2.30 GHz and a RAM of 4 GB. All the instances described in Section 3 has been used in the experimentation. The name of each instance has a number that represents the number of nodes it has. 30 executions have been run for each instance, and five different parameters are shown: average fitness value and its standard deviation, the median, the interquartile range and the average runtime (in seconds). These results can be observed in Table 2.

Besides, the fitness of the best solution found for each instance is shown in Table 3. Furthermore, the amount of objective function evaluations needed to reach these solutions are also represented, as well as the runtime.

## 6 Conclusions and future work

In this research a new multi-attribute TSP has been proposed, with the aim of addressing different sort of DRT problems. Concretely, the presented problem is a Multiple Asymmetric Traveling Salesman Problem with Simultaneous Pickup and Delivery. The objective of this problem is to find an exact number of routes, visiting all the nodes once, and only once, and minimizing the total traveling



Instance	Avg.	S. dev.	Median	I. R.	Time
MA-TSP-SPD_br17	66.1	1.4	65	2.2	0.71
MA-TSP-SPD_ftv33	1652.2	88.4	1575	149.2	1.35
MA-TSP-SPD_ftv35	1828.8	93.3	1765	123.5	1.46
MA-TSP-SPD_ftv38	1883.7	77.5	1813	120.7	1.64
MA-TSP-SPD_p43	5888.5	19.2	5873	24.7	1.69
MA-TSP-SPD_ftv44	2063.5	142.2	1896	260.7	1.83
MA-TSP-SPD_ftv47	2214.3	517.2	2235	186.0	2.23
MA-TSP-SPD_ry48p	18160.2	604.7	17784	601.5	3.09
MA-TSP-SPD_ft53	8614.5	444.8	8303	655.0	4.21
MA-TSP-SPD_ftv55	2239.6	141.6	2204	156.7	3.77
MA-TSP-SPD_ftv64	2505.9	145.1	2385	196.0	3.31
MA-TSP-SPD_ftv70	2720.5	136.7	2598	256.7	3.75
MA-TSP-SPD_ft70	44460.3	809.9	43717	1199.0	4.49
MA-TSP-SPD_kro124p	48277.6	2036.4	46407	3028.0	13.41
MA-TSP-SPD_ftv170	5482.5	309.6	5261	320.7	21.12
MA-TSP-SPD_rbg323	1851.3	59.8	1797	112.7	72.54
MA-TSP-SPD_rbg358	1856.3	72.6	1800	122.2	81.29
MA-TSP-SPD_rbg403	2859.2	50.6	2807	88.0	87.25
MA-TSP-SPD_rbg443	3121.4	55.7	3110	91.2	136.59

**Table 2.** Results obtained by the GB for the proposed MA-TSP-SPD. For each instance results average, standard deviation, median, interquartile range and time average are shown

cost. Furthermore, it is noteworthy that these traveling costs are asymmetric, and that two kinds of nodes coexist in the system: *delivery nodes* and *pickup nodes*.

Additionally, the recently proposed Golden Ball meta-heuristic has been used to solve the proposed benchmark composed by 19 instances. This benchmark is an adaption of the well-known ATSP benchmark that can be found in the TSPLib, and it has been previously used to solve other R-TSP problems. Finally, the solutions offered for the mentioned benchmark are considered the best ones, since it is the first time that the MA-TSP-SPD has been dealt in the literature.

As further work, it is intended to find another real-life situation, with a great social interest, with the aim of modeling them as a multi-attribute vehicle routing problem, and addressing them.

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Instance	Fitness	Evaluations	Time
MA-TSP-SPD_br17	65	40	0.65
MA-TSP-SPD_ftv33	1515	3783	1.72
MA-TSP-SPD_ftv35	1703	1939	1.71
MA-TSP-SPD_ftv38	1800	6351	2.28
MA-TSP-SPD_p43	5850	3421	1.53
MA-TSP-SPD_ftv44	1872	8887	3.57
MA-TSP-SPD_ftv47	2202	5047	2.07
MA-TSP-SPD_ry48p	17394	5157	2.14
MA-TSP-SPD_ft53	7901	8028	3.97
MA-TSP-SPD_ftv55	2001	5757	2.92
MA-TSP-SPD_ftv64	2323	7867	3.82
MA-TSP-SPD_ftv70	2540	16719	6.35
MA-TSP-SPD_ft70	43563	14190	5.26
MA-TSP-SPD_kro124p	45991	31664	13.85
MA-TSP-SPD_ftv170	5054	30779	21.20
MA-TSP-SPD_rbg323	1795	62850	80.72
MA-TSP-SPD_rbg358	1773	87121	112.56
MA-TSP-SPD_rbg403	2801	44375	81.67
MA-TSP-SPD_rbg443	3044	86175	163.57

**Table 3.** Best solutions found by the GB for the proposed problem

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